Design & Analysis of Rectangular Plate for Stress Relief

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***Abstract*—Almost all metallic structures consist of assembly of simple elements, which are connected to each other by connecting plates using bolts or rivets. These plates have hole to place the bolts or rivets; these make the structure weak and susceptible to the failure. Therefore, it is necessary to investigate the state of stress in the plate around the hole. It is very important to understand the behavior of the steel structure with holes or notches. The stress analysis of plate is done by using finite element method. Auxiliary holes are provided for stress relief. Around 35% of reduction in stress concentration in plate is observed by introducing auxiliary hole of optimum shape at optimum distance. Because of these auxiliary holes the flow of the tensile stresses will get smooth and it results in a increased life of that plate.**

***Index Terms*—Auxiliary hole, FEM, plate, stress, SCF**

I. INTRODUCTION

Structure strength is affected by any type of discontinuity. The flow of stress is altered due to discontinuity. Any discontinuity in a machine part changes the distribution of stress in the vicinity of the discontinuity so that the elementary stress equations can't be used to describe the state of stress in the part of their locations. Such discontinuities are referred to as stress raisers. Stress concentration may occur due to irregularities. The nominal stress generally exists if the member is free of stress raisers. The theoretical or geometrical stress concentration factor K_t is used to establish relationship between the actual maximum stresses at the discontinuity to the nominal stress. The nominal stress σ_{nom} is calculated by the elementary stress equations and the net area of the structure. The value of stress concentration factor depends the geometry of the structure. That is the particular material has no effect on the value of K_t .The analysis of geometric shapes to determine stress concentration factor is a difficult problem and not so many solutions can be found. The analytical treatment for such type of problem is not so easy, that's why the finite element method has been preferred for the analysis purpose. The stress concentration in structure changes with number of affecting factors. These factors include the size, the shape, the number, the location and relative size of the discontinuity in comparison with structure. It is required to use various techniques to reduce the effect of affecting factors as practicable to us. In this work to reduce stress concentration in the plate, approach used is provision of auxiliary holes on either side of the main hole.

II. METHODOLOGY

For the analysis of the plate having central circular hole FEM is used. This analysis is done by optistruct solver of the HYPERMESH software. The static analysis is implemented which is used to determine the displacement and stresses in the structure under linear static loading condition. The plates with diameter of central circular hole 10mm, 20mm, 30mm, 40mm, 50mm & 60mm are analyzed to find the relation in between diameter of hole and stress concentration factor of plate. For optimization of plate topology optimization is used. Using topology optimization one can find the best concept design that meets design requirement. The main purpose of design optimization is the creation of clear design of the structure which can define the requirement of material. we have a uniform use of material out of given set of isotropic or anisotropic material. After topology optimization auxiliary holes are provided on either side of main hole on the plate to reduce concentration factor. After compilations of results for design of experiments taguchi method is used to test the sensitivity of set of response variable to a set of control parameters by selecting appropriate orthogonal array to obtain the optimum setting of the parameters. Analysis of variance has done to find the percentage contribution of the controllable factors in the stress concentration. Finally the results obtained by finite element method are compared with the design of experiment results.

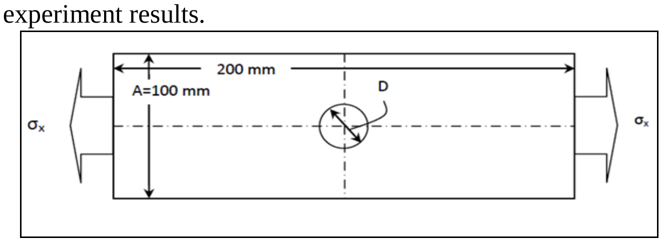

Fig.1.Model of Plate

A. Analysis of Plate

Initially six models of plate with different D/A ratio are prepared using CATIA software. This 3D CAD model is imported into HYPERMESH. The material model properties are defined. The HYPERMESH offer simple & easy mesh generation process. The boundary conditions are applied and the pressure of 6 MPa is applied. The analysis of the plate is done by applying 6 MPa. For the plates having D/A ratio 0.1, 0.2, 0.3, 0.4, 0.5 & 0.6 the on mises stresses has obtained. Same is obtained for plate having no hole.

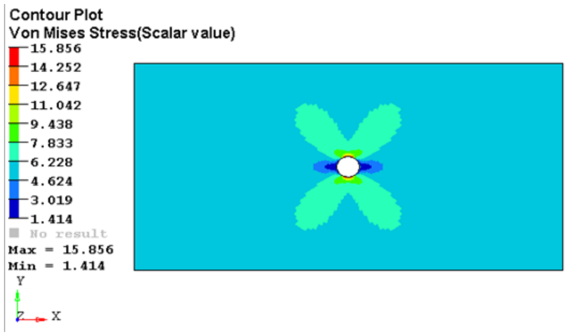

Fig.2.Von mises stresses for plate with D/A ratio =0.1

From the von mises stress & nominal stress the stress concentration factor for all six plates have calculated.

TABLE I
STRESS CONCENTRATION FACTOR OF PLATES

D/A	SCF
0.1	2.64
0.2	2.86
0.3	3.08
0.4	3.49
0.5	4.09
0.6	5.02
No Hole	1

The value of stress concentration factor of plate increases as D/A ratio of plate increases.

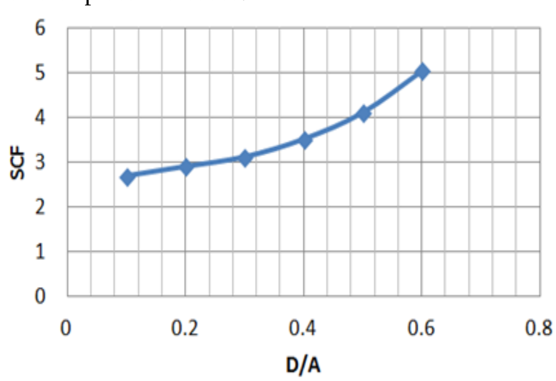

Fig.3.Region shown in iteration 48

III. TOPOLOGY OPTIMIZATION

Topology optimization is used to find the optimized material layout of the structure within specified boundary condition and the resulting layout satisfies all functional requirements s. The topology optimization helps us to find the region of plate where we can place the auxiliary holes without any interruption in original strength of plate. At 48th iteration of topology optimization optimized material layout of plate has obtained. It shows blue colored regions where we can place auxiliary holes for stress relief of plate. But the blue regions at the boundary of plate will change the shape of plate which is not desired. Hence auxiliary holes can be introduced at the blue ellipsoid region. That will not change shape of plate.

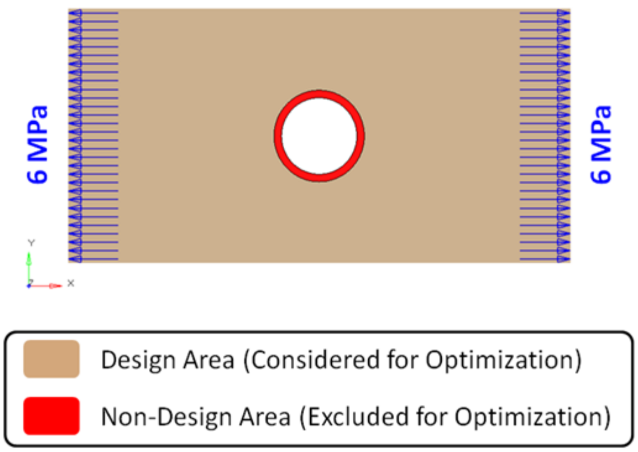

Fig.4 Optimization set up

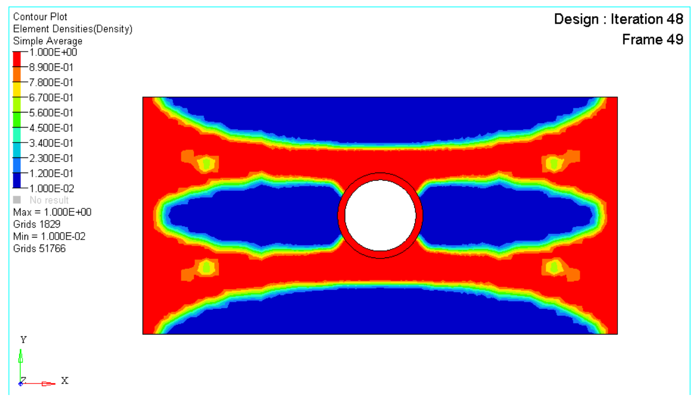

Fig.5.Region shown in iteration 48

A. Stress reduction with Auxiliary holes

Stress concentration can be reduced by placing auxiliary holes along with the main hole. The auxiliary holes can be placed in the ellipsoid region shown by last iteration of topology optimization. The minor diameter of that ellipsoidal shape is same as that of the diameter of the main hole at the centre of the plate and major axis is same as that of the t length of the rectangular plate. Hence the diameter of auxiliary hole depends on the ellipsoid shape obtained in the topology optimization. The size of auxiliary hole provided will be reduced as the distance between main hole and auxiliary hole increases. The distance of auxiliary hole from main hole is measured in the direction of length of plate along the circumference of circular hole. The plate models considered are having distances between main and auxiliary hole as 10mm, 20mm, 30mm, and 40mm. The trials are carried out for each plate model in the same manner of loading conditions. From analysis the maximum reduction of stress (17.21%) observed in the plate having D/A ratio 0.1 with auxiliary holes at 10 mm on either sides of the main hole & minimum reduction of stress (-6.33%) observed in the plate having D/A ratio 0.6 with auxiliary holes at 30 mm on either sides of main hole.

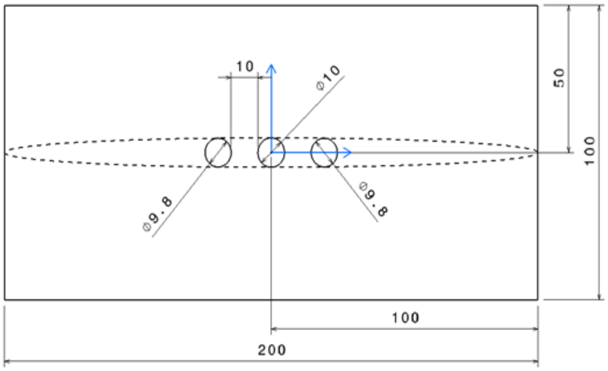

Fig.6. Plate with D/A=0.1 and distance=10

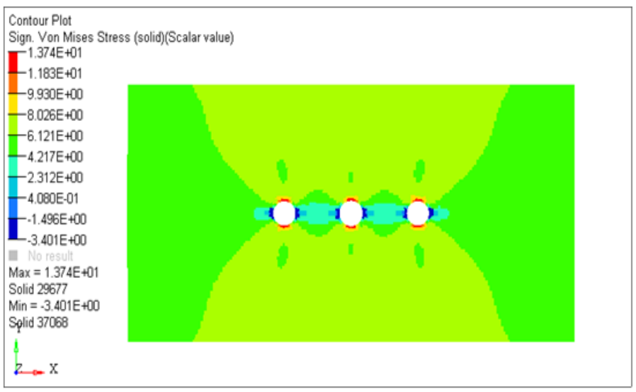

Fig.7.Von mises stresses for distance 10mm for D/A=0.1

The values of SCF for all plate models are plotted against the D/A ratio. The nature of graph is shown in figure below.

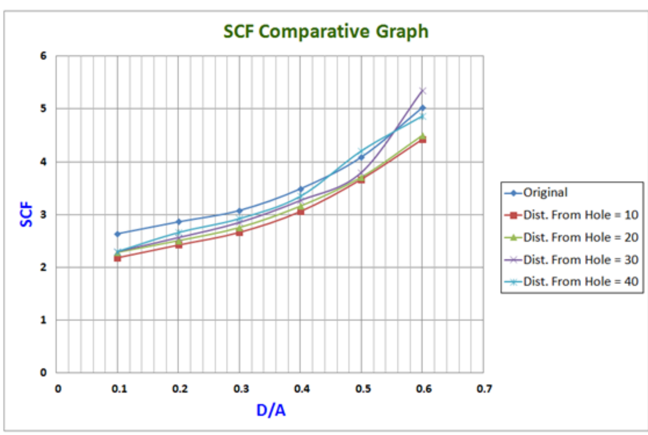

Fig.8. comparative graph for all trials

IV. DESIGN OF EXPERIMENTS

Design of experiments is the highly precise technique used for exploring what factor influence which feature. In the increasing competition in product quality, Design of experiments is very useful in improving quality of the product by statistical approach. This technique establishes existing cause effect relationship within the system. DOE is the technique which selects optimized parameters with high quality from specified range to create product which satisfy all functional requirements.

Taguchi methodology: The full factorial design results in large number of experiments. Taguchi methodology allows us to conduct experimentation on small set of experiments. It is performed by taking limited number of experiments from all large numbers of possible experiments. By using Taguchi analysis we can improve the accuracy of experiments. It finds the effect of different parameters on the mean as well as the variance of a process performance characteristic The Mean and variance define the process functioning. In the design of experiments by Taguchi method orthogonal arrays are used to arrange the control factors and their level of variations. The ANNOVA determines the individual percentage contribution of each and every factor that affects the response of process. For the required response optimum values can be obtained with minimum experimentation. It saves time & efforts.

Major steps followed for the Design of experiments using Taguchi method are as below:

A. *Establishment of objective function:*

The objective of the present work is to determine the optimum levels of the parameters configuring plate that result in the minimum stress concentration factor.

B. *Determination of controllable factors and their levels:*

In the present work all factors affecting the process quality are given in table II & their levels are given in table III below.

TABLE II
FACTORS AFFECTING THE PROCESS QUALITY

Control Factors	Noise Factors
Shape of Auxiliary hole	Material properties
Size of Auxiliary hole	Mechanical Vibrations
Distance of Auxiliary hole from main hole	Environmental conditions

TABLE III
LEVELS OF FACTORS

Factors	Levels		
	1	2	3
Shape of Auxiliary holes	Circle	Ellipse 1	Ellipse 2
Distance of Auxiliary hole from main hole	15	30	45

C. *Design of Taguchi orthogonal array layout:*

The Taguchi orthogonal array helps us to get complete information about the response affecting parameters with minimum experimentation work. As per number of controllable factors and their levels many orthogonal arrays are present. combinations of number of controllable factors. In the present work there are two controllable factors along with three levels. Hence L9 orthogonal array is selected for the present investigation. In L9 orthogonal array we have to perform 9 experiments.

TABLE IV
L9 ORTHOGONAL ARRAY WITH DETAILED VALUES

Expt. No.	Shape	Distance (mm)
1	Circle	15
2	Circle	30

3	Circle	45
4	Ellipse 1	15
5	Ellipse 1	30
6	Ellipse 1	45
7	Ellipse 2	15
8	Ellipse 2	30
9	Ellipse 2	45

D. Models of plates:

According to the combination of control factor with their specified level given by L9 orthogonal array as given in table 4, nine plate models are made. From the previous observations it is seen that plate having D/A=0.1 shows maximum reduction in SCF after introduction of auxiliary holes. So plate having D/A=0.1 is selected for further analysis. The diameter of main central circular hole is 10mm for this case. For the Ellipse 1 a/b ratio is 4 where a and b are its major and minor axis respectively. For the Ellipse 2 a/b ratio is 3. Plate models are made in AUTOCAD (2013). The diameter of auxiliary hole changes as per to the profile of the ellipsoid shape. As the distance between auxiliary hole and main hole increases the size of auxiliary hole decreases.

E. Analysis of Response:

The stress concentration factor obtained is given as the response. The results generated by the MINITAB 16 in terms of S/N ratio and mean are having large importance. The conclusion of the analysis can be drawn by taking reference of S/N ratio and mean. The Table below shows the S/N ratio and mean for all nine experiments conducted. Delta value given in the table shows the variations in mean of response within the various levels of factors. If the variation is more, then delta value will be more and therefore will be more contribution of that factor in the response. The rank defines order of contribution of each factor in the response. The delta ranks from higher value to lower value of all factors. In the manner order rank is given to all the control factors.

TABLE V
S/N RATIOS AND MEAN FOR RESPONSE

Expt. No.	Shape	Distance (mm)	SCF	S/N Ratio	Mean
1	Circle	15	1.79	-5.05706	1.79
2	Circle	30	1.95	-5.80069	1.95
3	Circle	45	2.02	-6.10703	2.02
4	Ellipse 1	15	1.71	-4.65992	1.71
5	Ellipse 1	30	1.94	-5.75603	1.94
6	Ellipse 1	45	1.96	-5.84512	1.96
7	Ellipse 2	15	1.85	-5.34343	1.85
8	Ellipse 2	30	1.83	-5.24902	1.83
9	Ellipse 2	45	2.03	-6.14992	2.03

TABLE VI
RESPONSE TABLE FOR SN RATIOS OF STRESS CONCENTRATION FACTOR

Column	Factors	Level 1	Level 2	Level 3	Delta	Rank
1	Shape	-5.655	-5.420	-5.581	0.235	2
2	Distance (mm)	-5.020	-5.602	-6.034	1.014	1

TABLE VII
RESPONSE TABLE FOR MEANS OF STRESS CONCENTRATION FACTOR

Column	Factors	Level 1	Level 2	Level 3	Delta	Rank
1	Shape	1.920	1.870	1.903	0.050	2
2	Distance (mm)	1.783	1.907	2.003	0.220	1

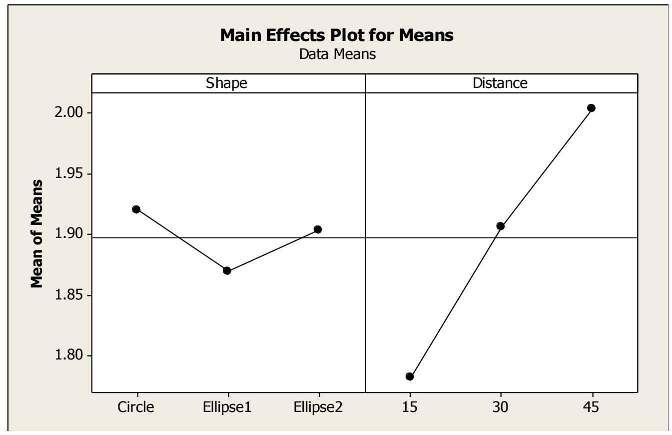

Fig.9.Main effect plot for means

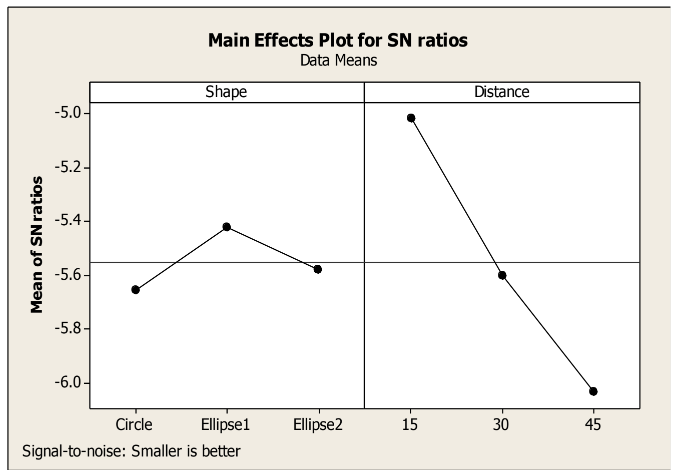

Fig.10. Main effect plot for S/N ratios

From the main effect plots it is clear that the stress concentration factor will minimum at level 2 for shape of auxiliary hole and level 1 for distance of auxiliary hole.

F. Percent contribution:

The optimum levels of controlling parameters are found out by using mean and S/N ratio analysis. Each factor is ranked as per their contribution in the response of the system. Now in order to find out significant factors involved in the analysis determination of their percent contribution F ratio is needed. The F ratio can be calculated by analysis of variance (ANOVA). The analysis of variance for stress concentration factor is shown in Table below.

TABLE VIII

ANOVA FOR STRESS CONCENTRATION FACTOR

Sr. No.	Factor	DOF	MSS	F ratio	% Contribution
1	Shape	2	0.0019	0.025	4.12
2	Distance (mm)	2	0.03648	0.48	77.15
3	Error	4	0.07534		18.73
4	Total	8			100

G. *Confirmation experiment:*

The final step of the Taguchi method is the confirmation experiments conducted for examining the quality characteristics. In order to verify the results obtained from Taguchi analysis, a confirmation experiment need to be performed. As per the steps of confirmation experiment the optimum combination of the levels for the plate parameters to minimize the stress concentration factor is shown below.

TABLE IX

OPTIMUM CONDITIONS OF PLATE PARAMETERS FOR DIFFERENT RESPONSES

Responses	Control Factors		Observed
	Shape	Distance	
SCF	Ellipse 1	15	1.71

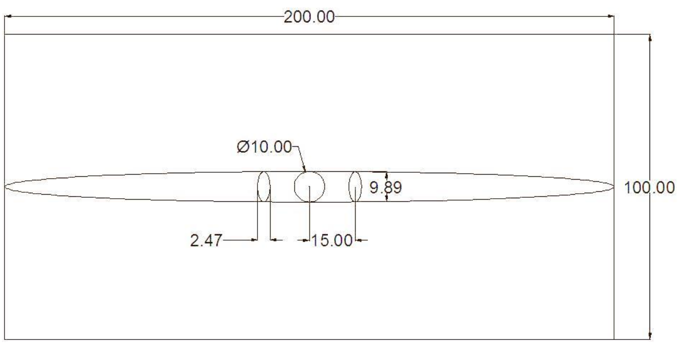

Fig.11. Model of plate at optimum condition

H. *Estimated Mean of all Responses:*

After getting optimal combination of process parameters and their levels we have to follow final step. The final step is to verification of the estimated result and experimental value. Estimated stress concentration factor at optimum condition is given as,

$$\hat{\mu} = \overline{A2} + \overline{B1} - \overline{T}$$

TABLE X

ESTIMATED STRESS CONCENTRATION FACTOR AT OPTIMUM CONDITION

Grand average	Shape	Distance	SCF
$\overline{T}$	$\overline{A2}$	$\overline{B1}$	
1.90	1.87	1.78	1.75

I. *Confidence Interval:*

The estimated results obtained from the experiments are the point estimate based averages of the results. It is good to have range of predicted results instead of only one value at the time of confirmation of experiment. Observed results should fall within between the range of predicted values with some confidence. Observed value should fall in between maximum value and minimum value Following statistical expression is used to find value of confidence interval.

$$C.I = \pm \sqrt{(F(1,n2) \times Ve) / Ne}$$

Where

F (1, n2) = F value of From F table at (1,n2) (For 95%)

Ne = Effective number of replications

$$\hat{\mu} - C.I \le \hat{\mu} \le \hat{\mu} + C.I$$

TABLE XI

COMPARISON BETWEEN EXPERIMENTAL RESULTS AND PREDICTED RESULTS

Sr. No.	Response	Predicted Results	Confidence Interval (C.I)		Observed results
			Lower	Upper	
1	SCF	1.75	0.99	2.51	1.71

From the above result table we can say that that the observed result is falling within the confidence interval range of predicted results.

V. CONCLUSION

From the finite element analysis the following findings are reported. From the analysis it is observed that the stress concentration factor in plate increases as the D/A ratio increases. The von mises stresses also increases as the D/A ratio increases. When auxiliary holes are placed in plate with main hole then reduction in stress concentration factor is observed. However increase in SCF is reported by some amount when auxiliary holes are placed at a distance of 40 mm in a plate with D/A ratio 0.5 and when they are placed at a distance of 30mm in a plate having D/A ratio 0.6. The maximum reduction in SCF is reported which is 17.21% in a plate having D/A ratio 0.1. Almost in every plate model the value of SCF increases as the distance of auxiliary hole from the main hole increases. So SCF is sensitive to D/A ratio and the distance of auxiliary hole from the main hole. From the Taguchi method the estimated value of SCF is 1.75.This value is very close to the FEM value of SCF which is 1.71 when ellipses having a/b ratio 4 are placed at a distance of 15mm from the main hole of plate having D/A ration 0.1. The auxiliary holes having ellipsoidal shape show better reduction in SCF than the circular shaped auxiliary holes. The observed results are falling within the confidence interval of predicted results hence the confirmation experiment has validated the result. The maximum reduction in SCF reported is 35.23%.

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References

Research Papers

[1] V.G.Ukadgaonker and D.K.N.Rao "A general solution for stresses around holes in symmetric laminates under in plane loading", Composite structure 49, pp 339-354 (2002).

[2] V. G. Ukadgaonker and Vyasraj Kakhandki, "Stress analysis for an orthotropic plate ith an irregular shaped hole for different in plane loading conditions-Part 1", Composite structures 70, pp 255-274,(2005).

[3] V.G.Ukadgaonker and D.K.N.Rao"A general solution for moments around holes in symmetric laminates" , Composite structures 49, pp 41-54, (2000).

[4] Zheng Yang, Chang Boo Kim, Chongdu Cho, Hyeon Gyu Beom,"The concentration of stress and strain in finite thickness elastic plate containing a circular hole" , International journal of solids and structures 45, pp 713-731 (2008).

[5] Shubhrata Nagpal, Dr. S.Sanyal, Dr. N.K.Jain, "Analysis and mitigation of stress concentration factor of a rectangular isotropic and orthotropic plate with central circular hole subjected to in plane static loading by design optimization", International Journal of Innovative research in Science, Engineering and Technology Vol. 2, No, Issue 7,(July 2013).

[6] C.G.Mekalke, M.V.Kavade, S.S.Deshpande, "Analysis of a plate with a circular hole by FEM", IOSR Journal of Mechanical and Civil Engineering, ISSN: 2278-1684, pp 25-30.

[7] Shubhrata Nagpal, S.Sanyal, Nitin Jain, "Mitigation curves for determination of relief holes to mitigate stress concentration factor in thin plates loaded axially for different discontunities", International Journal of Engineering and Innovative Technology, Volume 2, Issue 3, ISSN 2277-3754, (September 2012).

[8] William L.Ko, "Stress concentration around a small circular hole in the HiMAT composite plate", NASA Technical Memorandum 86038, (1985).

[9] B.Mallikarjun, P.Dinesh, K.I.Paashivamurthy, "Finite element analysis of elastic stresses around holes in plate subjected to uniform tensile loading", Bonefring International Journal of Industrial Engineering and Management Science, Volume 2, No. 4, (December 2012).

[10] D.B.Kawadkar, Dr.D.V.Bhope, S.D.Khamankar, "Evaluation of stress concentration in plate with cutouts and its experimental verification", International journal of engineering research and applications,ISSN 2248-9622,Volume 2,Issue 5, (September-October 2012).

[11] Shubhrata Nagpal, "Optimization of rectangular plate with central square hole subjected to in plane loading for mitigation of SCF", International Journal of Engineering Researh and Technology,ISSN 2278-0181,Volume 1,Issue 6 (August 2012).

Reference books

[12] "Design of machine elements" Edition 3 By V.B.Bhandari,Tata McGraw Hill Publication.

[13] "Design of machine elements" Edition 8 By J.F.Shigley,Tata McGraw Hill Publication.

[14] "Taguchi techniques for quality engineering" Phillip J.Ross McGraw Hill Publication