Analytical Modeling and Self-tuned Fuzzy-PID Logic based Control of Quarter Car Suspension System

Amit A. Divekar, PG student, Department of Mechanical Engineering, Sinhgad College of Engineering, Pune-411041.

Abstract— **A suspension system is responsible for driver comfort and safety. It isolates car body from road disturbances. A quarter car suspension is a suspension of $1/4^{th}$ car body mass. Suspension system has three types: passive, semi-active and active. A passive suspension has fixed damping rates determined by their design. Semi-active and active suspension uses selective damping rates. In active suspension, energy is added externally to change damping coefficient. This paper focuses on the development of model for 2 DOF quarter car and control algorithm in order to adjust damping rates of damper according to road disturbances. This paper proposes advanced Fuzzy Logic Controller (FLC) to minimize sprung mass displacement and Suspension Working Space (SWS) using MATLAB Simulink. The proposed fuzzy-PID algorithm has better performance in reducing sprung mass displacement than PID controller**

Index Terms— **Fuzzy Logic, PID, Fuzzy-PID, Quarter Car Suspension, Simulink, Semi-active suspension, MR Damper.**

I. INTRODUCTION

A well controlled suspension system will provide ride comfort to driver and passengers, road holding and vehicle handing, and good insulation from road vibrations. The typical or passive suspension is consisted of coil springs with fixed spring stiffness and dampers fixed damping coefficient. Due to fixed suspension settings, performance of suspension system is limited. Therefore, active and semi-active suspension systems are developed using variable damping coefficient damper and sensors to make optimization of sensor settings.

A semi-active suspension system has more performance than passive suspension and utilized energy less than active suspension system. Also the components of active suspension are costlier and therefore active suspension system is used in high end cars. Due to cost problem, a semi- active suspension system provides more affordable solution in comparison with active suspension. Failure of components of active suspension will stop functioning whole suspension system. But in semi-active suspension system will work as passive suspension system even its component stopped functioning due to failure. The semi-active suspension system is formed by the springs and damper with variable spring stiffness and damping coefficient. The damping coefficient can be adjusted based on the measured data from vehicle sensors. Such a suspension system provides effective improvement of comfort and handling for vehicles.

Zeinali and Mat Darus (2012) proposed fuzzy PI and PD controller to control the semi-active suspension system and showed effectiveness of fuzzy-PID controller to control suspension system in comparison with PID controller. Chen *et al* (2011) proposed a Sliding Mode Controller (SMC) for semi-active suspensions with non-linear model in order to improve suspension performance. Alvarez-Sanchez (2013) proposed sprung mass estimatation using algebraic method. Shiao *et al* (2010) developed self tuned fuzzy logic controller (FLC) to control magneto rheological (MR) damper and proposed variable stiffness damper and air sprigs to replace conventional coil spring. Tandel *et al* (2014) presented PID controller with various combination of damping coefficient and spring stiffness using the toolbox SimMechnics and Simulink of MATLAB software to improve suspension performance. Emam (2015) proposed and designed self tuned fuzzy PID controller to minimize suspension working space (SWS). Rashid *et al* (2011) developed and experimentally implemented hybrid FLC for MR based semi-active suspension system showed suitability of hybrid FLC system to reduce road vibrations in comparison with conventional PID controller. Anubi and Crane (2015) developed a semi-active suspension system for non-linear system with variable spring stiffness and variable damping coefficient and achieved variable stiffness by varying free length of spring**.** From literature review, it is found that fuzzy-PID controller is useful solution over classical PID controller. Many researchers have used error and rate of error as the inputs for fuzzy controller. In this paper, error and sprung mass velocity is used as inputs.

This paper compares self tuned fuzzy-PID controller with classical PID controller to minimize sprung mass displacement. The Section II presents mathematical modeling of quarter car and presents simulink model for passive suspension system. Section III explains controller design. Section IV discusses results. Section V presents conclusion.

II. MATHEMATICAL MODELING

Quarter car model is 1/4th part of the total car body. It consists of tire, shock absorber, sprung and unsprung mass bodies. Sprung mass is supported above suspension system. It includes the car body, frame, internal components, passengers etc. Unsprung mass is suspended below the part of suspension system. It consists of tire, rim, hub, bearings, knuckle etc. Figure 1 presents the quarter car model.

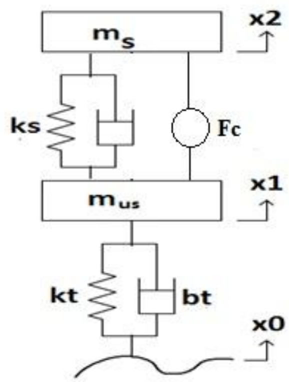


Figure 1. Quarter Car Suspension Model

Spring and damper characteristics are assumed to be linear for obtaining mathematical model. Lagrangian principle is used for deriving mathematical model of the system.

$$L = KE - PE \quad (1)$$

where, L= Lagrangian,

KE= Kinetic energy,

PE= Potential Energy,

Kinetic energy and potential energy is given as

$$KE = \frac{1}{2} m_s \dot{x}_2{}^2 + \frac{1}{2} m_{us} \dot{x}_1{}^2 \quad (2)$$

$$PE = \frac{1}{2} k_t (x_1 - x_0)^2 + \frac{1}{2} k_s (x_2 - x_1)^2 \quad (3)$$

Lagrangian is given as from equations (1), (2) and (3)

$$L = \frac{1}{2} m_s \dot{x}_2{}^2 + \frac{1}{2} m_{us} \dot{x}_1{}^2 - [\frac{1}{2} k_t (x_1 - x_0)^2 + \frac{1}{2} k_s (x_2 - x_1)^2] \quad (4)$$

$$P = \frac{1}{2} b_t (\dot{x}_1 - \dot{x}_0)^2 + \frac{1}{2} b_s (\dot{x}_2 - \dot{x}_1)^2 \quad (5)$$

where, P= Dissipative Energy

$$F_c = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} + \frac{\partial P}{\partial \dot{x}} \quad (6)$$

The dynamics of equations for suspension system are given as by using Lagrangian method:

$$m_s \ddot{x}_2 = k_s (x_1 - x_2) + b_s (\dot{x}_1 - \dot{x}_2) + F_c \quad (7)$$

$$m_{us} \ddot{x}_1 = k_t (x_0 - x_1) + b_t (\dot{x}_0 - \dot{x}_1) - k_s (x_1 - x_2) - b_s (\dot{x}_1 - \dot{x}_2) - F_c \quad (8)$$

where, m_s= sprung mass,

m_{us}= unsprung mass,

k_s= spring stiffness of shock absorber,

k_t= spring stiffness of tire,

b_s = damping coefficient of shock absorber,

b_t = damping coefficient of tire,

x_0 = road displacement,

x_1 = tire displacement,

x_2 = sprung mass displacement.

F_c = controlled damping force.

The state space model for quarter car suspension can be obtained from equations (7) and (8) and given as

$$\begin{bmatrix} \dot{x}_2 \\ \ddot{x}_2 \\ \dot{x}_1 \\ \ddot{x}_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k_s}{m_s} & \frac{-b_s}{m_s} & \frac{k_s}{m_s} & \frac{b_s}{m_s} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_{us}} & \frac{b_s}{m_{us}} & \frac{-(k_s + k_t)}{m_{us}} & \frac{-(b_s + b_t)}{m_{us}} \end{bmatrix} \begin{bmatrix} x_2 \\ \dot{x}_2 \\ x_1 \\ \dot{x}_1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{m_s} \\ 0 & 0 & 0 \\ \frac{k_t}{m_{us}} & \frac{b_t}{m_{us}} & \frac{-1}{m_{us}} \end{bmatrix} \begin{bmatrix} x_0 \\ \dot{x}_0 \\ F_c \end{bmatrix} \quad (9)$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_2 \\ \dot{x}_2 \\ x_1 \\ \dot{x}_1 \end{bmatrix} \quad (10)$$

Table 1 shows system parameters used for simulation:

Table 1. Parameters of Quarter Car [3]

Parameter	Value	Unit
m_s	208	Kg
m_{us}	28	Kg
k_s	18709	$\frac{N}{m}$
b_s	1300	$\frac{N.s}{m}$
k_t	127200	$\frac{N}{m}$
b_t	10	$\frac{N.s}{m}$

Figure 2 shows simulink model for passive suspension system. This model is simulated for 0.08 step input as road profile.

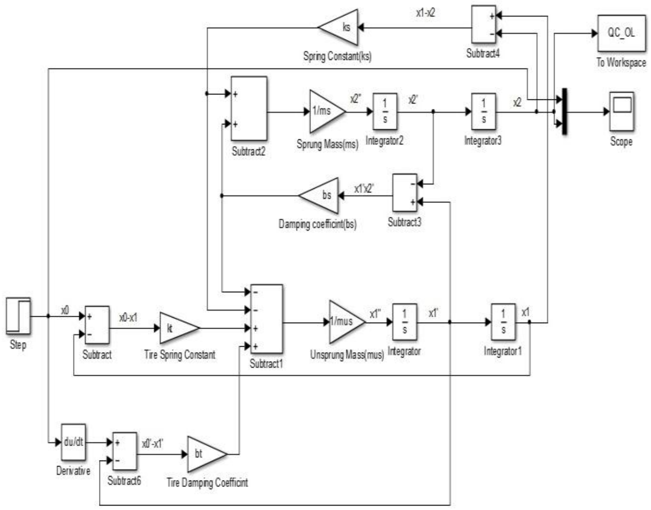


Figure 2. Simulink model for passive suspension system

III. CONTROLLER DESIGN

This section presents design of PID and fuzzy-PID controller for suspension system.

A. *PID Controller*

The PID controller is the most common and extensively used controller. Here it is used to control damping force of suspension system. Most feedback loops are controlled by PID algorithm or minor variations of it. A PID controller calculates an error value as the difference between a desired set point and a measured variable. The controller makes an effort to minimize this error by adjusting the process control inputs. The PID controller has three separate constant parameters: the proportional, the integral and derivative values, denoted K_p, K_i and K_d respectively. Proportional control improves rise time. Integral control eliminates steady state error. Derivative control improves overshoot. The algebraic sum of these three actions is used to control the process through a control actuator such as the current of MR damper or flow of hydraulic actuator.

$$f_c = u = K_p(1 + \frac{1}{T_i}\int edt + T_d\frac{de}{dt}) \quad (11)$$

$$f_c = u = K_p + K_i\int edt + K_d\frac{de}{dt} \quad (12)$$

where, f_c= Controlled damping force,

K_p = Proportional gain,

T_i = Integral time,

T_d= Derivative Time,

K_i = Integral gain = $\frac{K_p}{T_i}$,

K_d = Derivative gain = $K_p * T_d$.

Figure 3 shows the PID controller applied for suspension system. Saturation limits: 800 N to -800 N are used to prevent actuator wind-up and limit controller force. The PID controller is tuned using ZN method. Calculated parameters are shown as below:

K_p = 4.3794,

K_i = 92.39,

K_d =200.

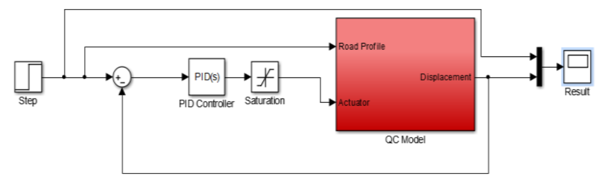


Figure 3. Suspension System with PID controller

B. *Fuzzy-PID Controller*

A FLC involves of three important functions. These three fuctions are

- Fuzzification: The input crisp data are converted into fuzzy data sets. Fuzzy data sets are represented with appropriate membership functions.
- Inference Engine: These fuzzy data sets are then defined using “IF-THEN” rules.
- Defuzzification: Solved data by” IF-THEN” rules are converted into crisp data.

The proposed fuzzy PID controller determines variables K_p , K_i and K_d by self tuning using fuzzy logic controller. Fuzzy logic controller is designed for suspension system using two inputs and three outputs. Two inputs are difference between set point and measured output parameter and sprung mass velocity. Three outputs parameters are K_p, K_i and K_d, which determines controlled force. Figure 4 shows the design of fuzzy-PID controller. Figure 5 shows implementation of fuzzy-PID controller for suspension system.

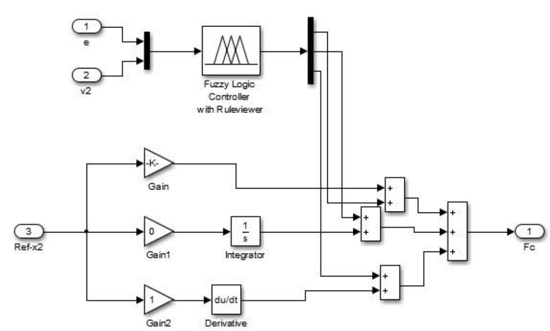


Figure 4. Fuzzy-PID Controller

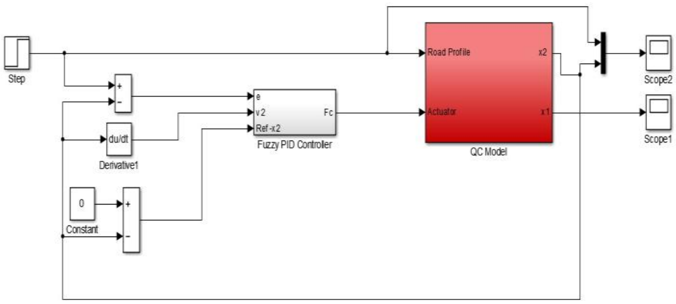
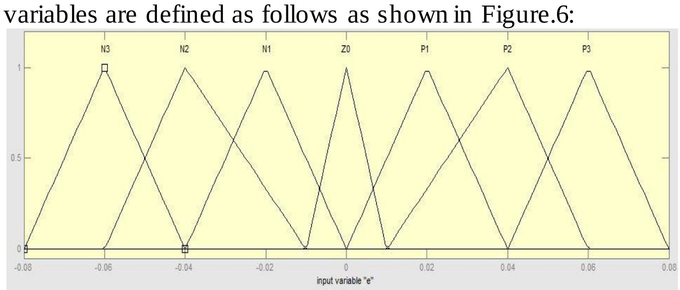
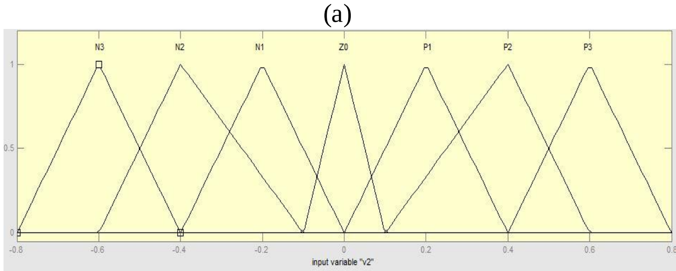


Figure 5. Suspension system with Fuzzy-PID controller

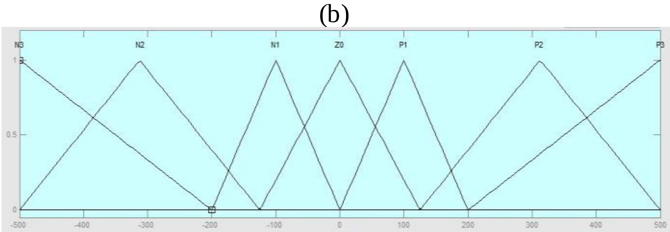
Membership functions for input variables and output variables are defined as follows as shown in Figure.6:



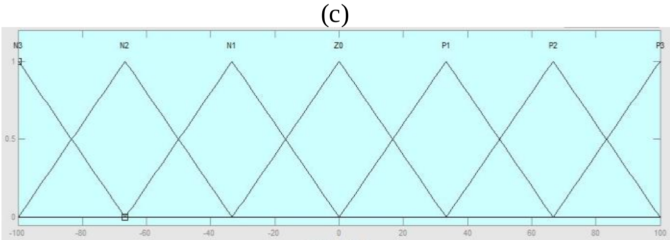
(a)



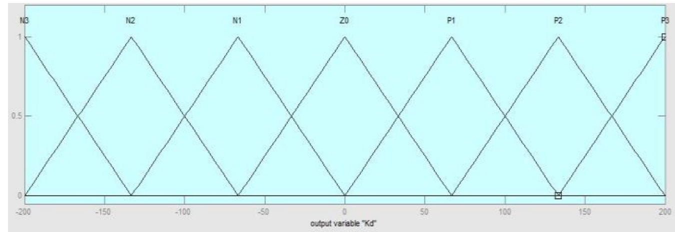
(b)



(c)



(d)



(e)

Figure 6. Membership functions for (a) Input variable e (b) Input variable V_2 (c) Output variable K_p (d) Output variable K_i (e) Output variable K_d

Table 2 shows the IF-THEN rule used for fuzzy logic. Total 49 rules has been defined using inputs, error (e) and sprung mass velocity (v2) for output parameters: K_p, K_i and K_d.

Table 2. Fuzzy logic rule

e \ v2	N3	N2	N1	Z0	P1	P2	P3
N3	P3	P3	P3	P3	P2	P1	Z0
N2	P3	P3	P3	P2	P1	Z0	N1
N1	P3	P3	P2	P1	Z0	N1	N2
Z0	P3	P2	P1	Z0	N1	N2	N3
P1	P2	P1	Z0	N1	N2	N3	N3
P2	P1	Z0	N1	N2	N3	N3	N3
P3	Z0	N1	N2	N3	N3	N3	N3

III. SIMULATION RESULTS

The model with PID controller and self tuned fuzzy-PID controller are simulated using step input of 0.08 m. The results of simulation are compared. Figure 7, 8, 9 shows sprung mass displacement, velocity and acceleration of car body respectively. From results, fuzzy-PID controller has better performance to reduce sprung mass displacement in comparison with PID controller.

Table 3 Characteristics for Suspension System

Sr. No.	Characteristics	Passive	PID	Fuzzy-PID
1.	Rise time	0.117sec	0.1071sec	0.112 sec
2.	Settling time	1.515 sec	1.348 sec	1.018 sec
3.	Overshoot	51.10%	45.62%	9.95%
4.	Steady state	2.969sec	2.5660sec	1.352sec

Table 3 shows that fuzzy-PID has 78.18 % less overshoot and 24.48 % better settling time in comparison with PID controller. Intitialy, fuzzy-PID requires more time to fire mebership functions to determine K_p, K_i and K_d parameters. So intially, fuzzy-PID has small delay. Therfore fuzzy-PID controller has more rise time than classical PID controller.

Also fuzzy PID controller has less sprung mass accleration and less sprung mass velocity.

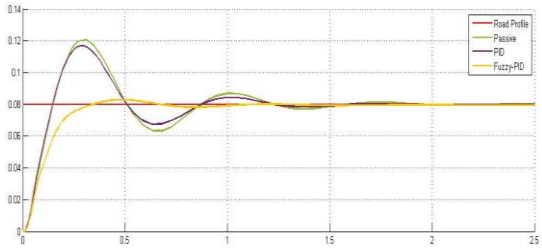


Figure 7. Comparison of Sprung Mass Displacement

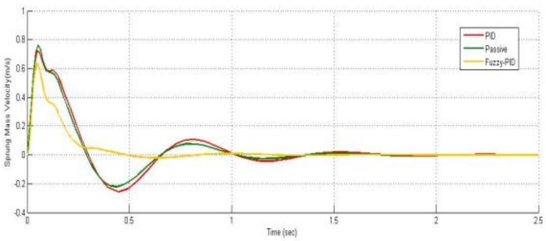


Figure 8. Comparison of Sprung Mass Velocity

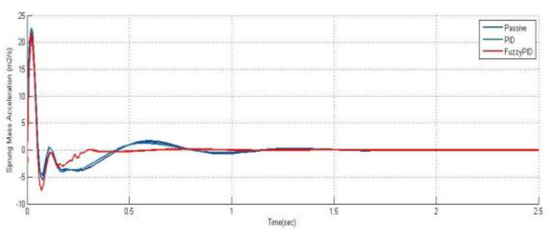


Figure 9. Comparison of Sprung Mass Acceleration

IV. CONCLUSION

The proposed Fuzzy-PID controller has better results to damp road vibrations than conventional PID controller in order to limit sprung mass displacement. Disadvantage of using fuzzy-PID controller is the time required for computation. Though fuzzy- PID controller can be used to control suspension system for better performance. Fuzzy-PID has less overshoot and settling time in comparison with passive and PID controller. PID controller gets saturated and so saturation limits of +800N to -800N are applied. The main advantage of using fuzzy and fuzzy-PID is that there is no actuator saturation.

ACKNOWLEDGMENT

I wish to express my warm and sincere thank to Dr. Jayesh L. Minase, for his invaluable guidance along with the care extended throughout the project work. I thank SCOE, Pune for allowing me to use various infrastructural facilities and resources.

REFERENCES

[1] Mohammadjavad Zeinali and Intan Zaurah Mat Darus, "Fuzzy PID Controller Simulation for a Quarter-car Semi-active Suspension System using Magnetorheological Damper", *IEEE Conference on Control, Systems and Industrial Informatics (ICCSII), Bandung, Indonesia*, pp. 104-108 , 2012.

[2] Bo-Chiuan Chen, Yu-Hua Shiu and Feng-Chi Hsieh, "Sliding-mode Control for Semi-active Suspension with Actuator Dynamics", *Vehicle System Dynamics Vol.* 49, pp. 277- 290, 2011.

[3] Ervin Alvarez-Sánchez, "A Quarter-car Suspension System: Car Body Mass Estimator and Sliding Mode Control", *Procedia Technology* 7, pp. 208- 214, 2013, doi: 10.1016/j.protcy.2013.04.026.

[4] Yaojung Shiao, Chun-Chi Lai and Quang-Anh Nguyen, "The Analysis of a Semi-Active Suspension System", *SICE Annual Conference, Taipei, Taiwan* pp. 2077– 2082, 2010.

[5] Anand Tandel, A. R. Deshpande, S. P. Deshmukh and K. R. Jagtap "Modeling, Analysis and PID Controller Implementation on Double Wishbone Suspension using SimMechanics and Simulink", *Procedia Engineering* 97, pp. 1274 –1281, 2014, doi: 10.1016/j.proeng.2014.12.406.

[6] A.S. Emam, "Fuzzy Self Tuning of PID Controller for Active Suspension System", *Advances in Powertrains and Automotives, Vol.* 1, pp. 34 –41, 2015.

[7] Muhammad Mahbubur Rashid, Nasrudin A. Rahim, Mohammed Azlan Hussain, and M. Azizur Rahman, "Analysis and Experimental Study of Magnetorheological-Based Damper for Semiactive Suspension System Using Fuzzy Hybrids", *IEEE Transactions on Industry Applications, Vol.* 47, *No.* 2, pp. 1051-1059, 2011.

[8] Olugbenga Moses Anubi and Carl Crane, "A New Semiactive Variable Stiffness Suspension System Using Combined Skyhook and Nonlinear Energy Sink-Based Controllers", *IEEE Transactions on Control Systems Technology, Vol.* 23, *No.* 3, pp.937-947, 2015.