Contact Stress Analysis of Composite Spur Gear Using FEM and Hertz Theory

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Abstract—**Gears have a wide variety of applications from watches to very large mechanical units like lifting devices and automotives. Gear teeth normally fail when load is increased above certain limit. Therefore it is required to explore alternate material for gear manufacturing. Composite materials provide adequate strength with weight reduction and they have emerged out as a better replacement for metallic gears. The contact stress in the mating gear is the key parameter in gear design. This paper represent contact stress analysis of steel gear and composite gear using Hertz equation and by Finite Element Analysis using Ansys 16.0 Workbench. When compared, the results of both theoretical method and FEA show a good degree of agreement with each other.**

Index Terms—**Contact Stress, Finite element Analysis, Hertz Equation, Spur gear**

I. INTRODUCTION

THE rapid growth of heavy industries such as vehicle, shipbuilding and aircraft industries require extensive application of gear technology. Spur gear is a cylindrical shaped gear in which the teeth are parallel to the axis. Spur gears are easy to manufacture and it is mostly used to transmit power from one shaft to another shaft up to certain distance & it is also used to vary the speed & Torque. e.g. Watches, gearbox etc. The replacement cost of of spur gear is very high and also the system down time is one of the effect in which these gears are part of system. Failure of gear causes breakdown of system which runs with help of gear. E.g. automobile vehicle. So it becomes very important to increase the strength of gear to avoid the failure

Composites provide much improved mechanical properties such as greater strength to weight ratio, increase in hardness, and hence less chances of failure. So this work is concerned with replacing metallic gear with gear of composite material of Aluminium Silicon Carbide so as to improve performance of machine and to have longer working life. P.B. Pawar has developed a metal matrix composite of Aluminum based Silicon Carbide [1]. The composition of Silicon Carbide is varied in Aluminum and mechanical tests were performed. They proposed to use this material for power transmitting element like gears. Author P.B. Pawar has manufactured the spur gear from composite material of Aluminum Silicon Carbide. He has done FEA using Ansys 14.0 and concludes that composite gears offer improved properties over steel alloys and can be used as alternative for replacing metallic gears [2]. Neelima Devi and co authors worked on the mechanical characterization of Aluminum Silicon Carbide [3]. They found that the weight to strength ratio for composite is about three times that of mild steel and it is two times less in weight than aluminum of same dimension. Seok-Chul Hwang presents a contact stress analysis for tooth in contact of gears during rotation [4]. Contact stress analysis for spur and helical gears is carried out between two gear teeth at different contact positions during rotation. The variation of contact stress values during rotation is compared with the values of contact stress at the lowest point of single tooth contact. Ali Raad Hassan has developed a program to plot paired teeth in contact [5]. This program was run each 30 of rotation of pinion to create 10 cases. The program gave graphic results for different FE models and stress analysis was carried out in ANSYS. Sushil Kumar Tiwari found out the contact stress and bending stress for involute spur gear teeth in meshing by finite element method and the results are checked with those obtained by Lewis formula, Hertz equation and AGMA/ANSI equations [6]. They observed that Hertz theory is the primary basis of contact stress calculation and for determining bending stress in a pair of gear, Lewis formula is used. Vivek Karaveer has done the modelling of FEA of spur gear using ANSYS 14.5. He has compared the stress values and deformation for steel and grey cast iron [7].

II. DESIGN OF GEAR

The material properties of steel and Aluminium Silicon Carbide composite are given in the table 1.

TABLE I
MATERIAL PROPERTIES OF GEAR MATERIALS

Material Property	Steel	AlSiC
Young's Modulus	210 GPa	150 GPa
Poisson's Ratio	0.3	0.3
Ultimate tensile strength N/mm^2	200	151

The comparative study of steel gear and composite gear is done. So the basic design of spur gear is same for both the gears. The various parameters of gear design are given in the table below.

TABLE II
GEAR DESIGN PARAMETERS

Parameter	Gear Pair
No. of teeth	20
Gear Ratio	1
Module	4.5
Pressure angle	20
Pitch diameter	90
Face width	45
Center Distance	90
Torque (Nm)	302
Speed (rpm)	1000

III. THEORETICAL ANALYSIS OF CONTACT STRESS USING HERTZ THEORY

Earle Buckingham (1926) have used theory of Hertz to calculate the contact stress between a rotating pair of teeth while transmitting power by treating the pair of teeth in contact as cylinders of radii equal to the radii of curvature of the mating involutes at the pitch point. According to Hertz theory, when two cylinders are pressed together, the contact stress is given by

$$\sigma_c = \frac{2P}{\pi BL} \quad (1)$$

$$B = \sqrt{\left[\frac{2P\left(\frac{1-\mu_1^2}{E_1}+\frac{1-\mu_2^2}{E_2}\right)}{\pi l\left(\frac{1}{d_1}+\frac{1}{d_2}\right)}\right]} \quad (2)$$

Where,

σ_c = maximum value of contact stress (N/mm2)

P = force pressing the two cylinders together (N)

B = half width of deformation (mm)

L = axial length of cylinders (mm)

d_1, d_2 = diameters of two cylinders (mm)

E_1, E_2 = modulii of elasticity of two cylinder materials (N/mm2)

μ_1, μ_2 = poisson's ratio of the two cylinder materials (unitless)

Substituting the value of half width of deformation *B*, in equation (1) & squaring both sides we get,

$$\sigma_c^2 = \frac{1}{\pi}\left(\frac{P}{L}\right)\left[\frac{\left(\frac{1}{r_1}+\frac{1}{r_2}\right)}{\left(\frac{1-\mu_1^2}{E_1}+\frac{1-\mu_2^2}{E_2}\right)}\right] \quad (3)$$

If we treat the material of both the cylinders as same, then the modulus of elasticity and poisson's ratio will be equal. Therefore substituting $E_1 = E_2 = E$ and $\mu_1 = \mu_1 = \mu$ in equation (3) we get

$$\sigma_c^2 = \frac{1}{2\pi}\left(\frac{P}{L}\right)\left[\frac{\left(\frac{1}{r_1}+\frac{1}{r_2}\right)}{\left(\frac{1-\mu}{E}\right)}\right] \quad (4)$$

Now applying this equation to a pair of spur gear teeth in contact, we need to replace the radii *r1* & *r2* by the radii of curvature at the pitch point

$$r_1 = \frac{d_{pp}\sin\phi}{2} \text{ and } r_2 = \frac{d_{pg}\sin\phi}{2}$$

Now, since the pinion and gear have equal geometry in all respects as given in table 2, we have,

$$d_{pp} = d_{pg} = d_p$$

$$r_1 = r_2 = r = \frac{d_p \sin\phi}{2} \quad (5)$$

From (4) and (5) we get

$$\sigma_c^2 = \frac{1}{\pi(1-\mu)}\left(\frac{PE}{Lr}\right) \quad (6)$$

For same material of pinion and gear as stainless steel, then for stainless steel, from table 1 poisson's ratio μ =0.3.

$$r = r_p \sin\phi \text{ and } P = \frac{P_t}{\cos\phi}$$

The length *L* is same as the face width *b* of spur gears, therefore replacing *L* by *b* in equation. Substituting this value of in equation (6) and solving we have

$$\sigma_c = 0.6747\left[\frac{P_t E}{br_p \sin\phi\cos\phi}\right]^{\frac{1}{2}} \quad (7)$$

The tangential load acting on the tooth can be obtained by

$$P_t = \frac{2T}{d_p} = \frac{2\times 302}{90} = 594.54N$$

Putting the value of young's modulus E, face width b, pitch circle radius r_p and pressure angle in equation (7) , we get the value of contact stress:

For Steel

$$\sigma_c = 0.6747\left[\frac{594.5\times 210\times 10^3}{45\times 45\sin 20\cos 20}\right]^{\frac{1}{2}} = 54.62\frac{N}{mm^2}$$

For Composite material

$$\sigma_c = 0.6747\left[\frac{594.5\times 150\times 10^3}{45\times 45\sin 20\cos 20}\right]^{\frac{1}{2}} = 41.23\frac{N}{mm^2}$$

IV. FINITE ELEMENT ANALYSIS

Finite Element Method is the easy technique as compared to the theoretical methods to calculate the stress developed in teeth of gears. Therefore FEM is widely used for the stress analysis of mating gears. FE analysis is done in ANSYS Workbench 16.0 to determine the maximum contact stresses for steel and composite material. Also the deformation is found out for both the gears. CAD model of gear is created in CREO 2.0. It is imported as a IGES file in ANSYS 16.0.

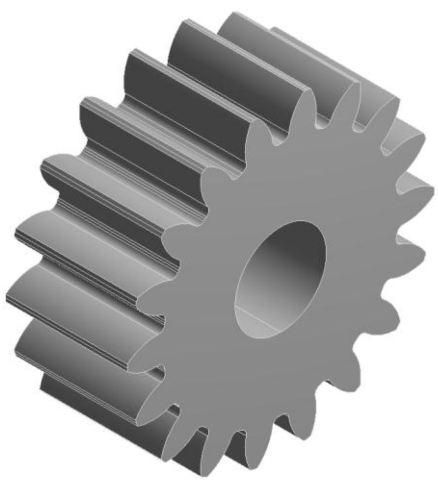

Fig. 1. CAD model of gear

A. Meshing

Meshing is done using Hexagonal mesh with number of elements of 499900 and number of nodes of 2125654. The element size is 0.8 mm. This mesh is used as it is fine and gives least number of elements with good results. So the calculation time is reduced.

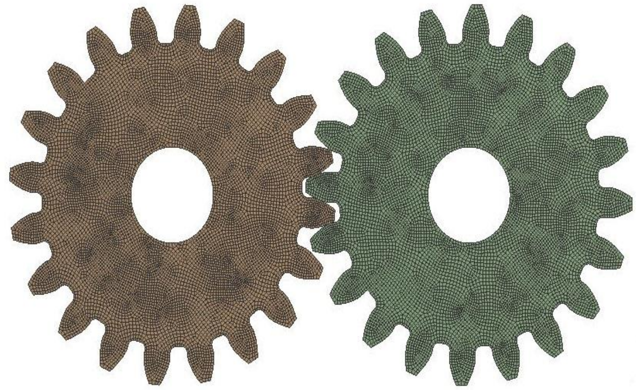

Fig.2. Mesh Model of gear

B. Boundary Conditions

Fixed support is applied on inner rim of the gear. Frictionless support is applied on the inner rim of pinion to allow its tangential rotation. Moment of 302Nm is applied on the surface of second gear.

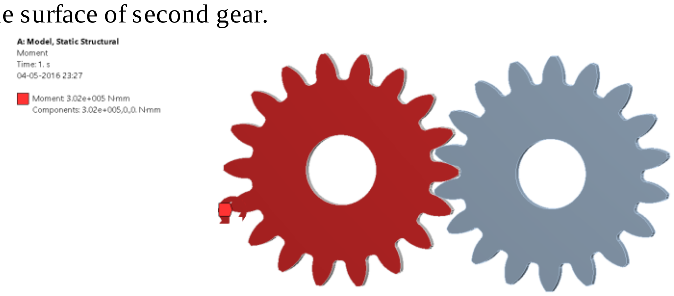

Fig. 3. Boudary Conditions of Ansys

V. RESULT AND DISCUSSION

Contact stress for steel and aluminium silicon carbide is calculated in ANSYS 16.0. Figure 4 shows the contact stress for steel which gives a stress of 52.14 MPa. Figure 5 shows contact stress for composite gear which gives a stress value of 39.18 MPa. Figure 6 presents the comparison of stress of steel and composite gear in bar chart form.

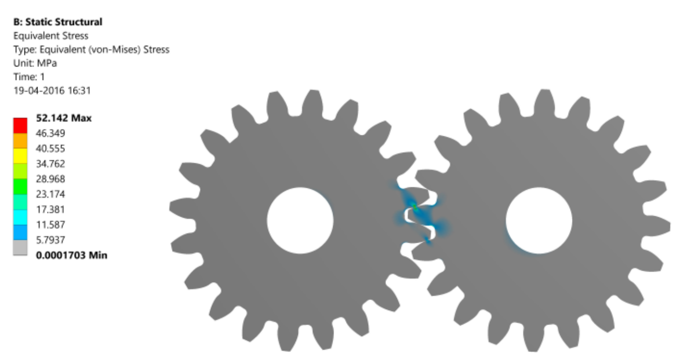

Fig. 4. Stress Diagram of Steel

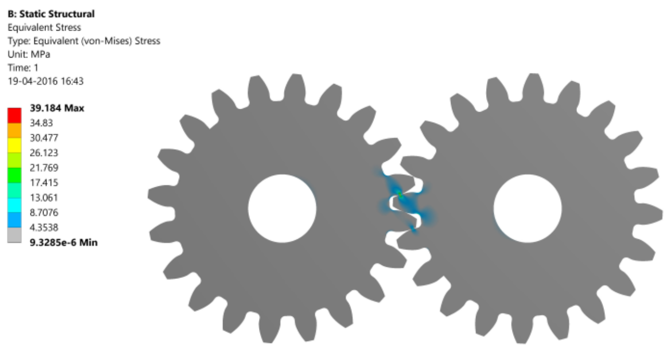

Fig. 5. Stress Diagram of Aluminium Silicon Carbide

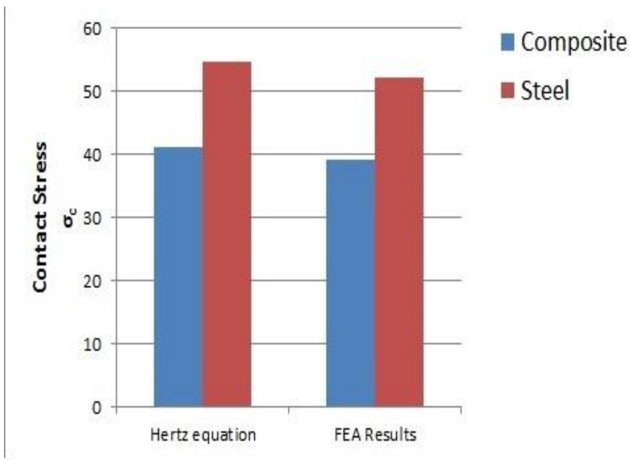

Fig. 6. Stress Comparison Analytical and FEA Method

Percentage difference of stress for steel and composite material using analytical method and FEA is calculated. It is shown in table 3.

TABLE 3

PERCENTAGE DIFFERENCE BETWEEN ANALYTICAL AND FEA

	Steel	Composite Material	% Reduction in Stress
Hertz Equation	54.62	41.38	24.24
FEA Method	52.14	39.18	24.87
% Difference	4.54	4.83	

VI. Conclusion

Here the theoretical maximum contact stress is calculated by Hertz equation. Also the FE analysis of spur gear is done to determine the maximum contact stress by ANSYS 16.0. It was found that the results from both Hertz equation and Finite Element Analysis are comparable. The results are well within the difference of 5%. Also it is observed that stress is reduced by nearly 25% due to the use of composite material.

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