Vibration Analysis of Cracked Beam: A Comparative Study

Miss Priyanka P. Gangurde [1], Dr. Santosh N. Shelke [2], GES's R.H.S.C.O.E.Nashik

Abstract— **The beam undergoes different kinds of loading which causes cracks in the beam. These cracks and their location effect changes the natural frequency and mode shapes of the beam. In the current work the natural frequency of cracked and uncracked beam having one end fixed and other is simply supported is investigated theoretically, numerically by using ANSYS software. Its experimental analysis is done by FFT analyzer. Structural Steel and Aluminum are taken as beam material. The cracked beam having crack depth 1mm, 1.5mm and 2mm are considered. Also different crack locations and crack shapes like triangular, rectangular and circular are considered and results are compared with the beam having no crack.**

***Index Terms*—ANSYS, Cracked beam, FFT, Fixed end, Mode shape, Natural frequency.**

I. INTRODUCTION

Vibration refers to mechanical oscillations about an equilibrium point. The oscillations may be periodic such as the motion of a pendulum or random such as the movement of a tire on a gravel road. A crack in a structural member introduces local flexibility that would affect vibration response of the structure. It is used to detect existence of a crack together its location and depth in the structural member. The presence of a crack in a structural member alters the local compliance that would affect the vibration response under external loads. However, when the displacements are large, linear beam theory fails to accurately describe the dynamic characteristics of the system. These large displacements cause geometric and other nonlinearities to be significant. The nonlinearities couple the modes of vibration and can lead to modal interactions where energy is transferred between modes.

Tarsicio Belendez et al. [3] have presented numerical and experimental analysis of a cantilever beam-a laboratory project to find out the geometric nonlinearity. In this paper he found out the deflection of a mild steel cantilever beam, under the action of a uniformly distributed load which is considered as its own weight. Paper presents the differential equation governing the behavior of this system which is difficult to solve due to nonlinear term. The experiment described in this paper is used to find out this geometric nonlinearity. Finally numerical result is find out by ANSYS software and compared with the experimental results which shows good agreement.

J. FernadNdez-Sad Ez et al. [4] has formulated approximate calculation of the fundamental frequency for bending vibration of cracked beam. A simplified method of evaluating the fundamental frequency for the bending vibrations of cracked Euler Bernoulli beams is presented. The method is based on the well-known approach of representing the crack in a beam through a hinge and an elastic spring, but here the transverse deflection of the cracked beam is constructed by adding polynomial functions to that of the uncracked beam. With this new admissible function, which satisfies the boundary and the kinematic conditions, and by using the Rayleigh method, the fundamental frequency is obtained. This approach is applied to simply supported beams with a cracked section in any location of the span. For this case, the method provides closed-form expressions for the fundamental frequency. Its validity is confirmed by comparison with numerical simulation results. In all the cases considered in this paper, the results are very close to those obtained numerically by the finite-element method.

Dr. Ravi Prasad et al. [5] has presented a paper on dynamic characteristics of structural materials using modal analysis. He carried out experimental analysis of cantilever beams made with different materials such as Brass, Aluminum, Steel, Copper using vibration analyzer for free vibration. The FRFs were obtained using OROS vibration analyzer which is proceeds to find out the various dynamic characteristics such as natural frequency, damping coefficient and mode shapes of the beam.

Ranja Behra et al.[6] has analyzed Aluminum cantilever beam specimen with & without crack having inclined crack at different crack location & crack depth experimentally on FFT & validation is done by finite element method. It is found that in first mode shape the amplitude decreases with increase in location from fixed end but in second and third mode shapes the amplitude increases with increase in location from fixed end at constant crack depth and constant crack inclination angle of the cracked cantilever beam. Moreover at particular location in the beam amplitude decreases with increase in crack depth in case of first mode shape, but amplitude increases with increase in crack depth in case of second and third mode shapes of the cracked beam at constant crack inclination angle.

P. Yamuna et al. [7] published a paper on vibration analysis of beam with varying crack location .The objective of this study is to analyze the vibration behavior of a simply supported beam using FEM software ANSYS subjected to a single triangular crack under free vibration. Material properties of steel are considered for the simply supported beam. Besides this, information about the variation in location and depth of cracks in cracked steel beams is obtained using this technique. It can be found that at symmetric positions of the crack position of the beam the lowest fundamental frequencies have almost equal value.

.

II.RESEARCH GAP

The following graph shows the percentage variation of research work on types of beam use for vibration analysis of cracked beam.

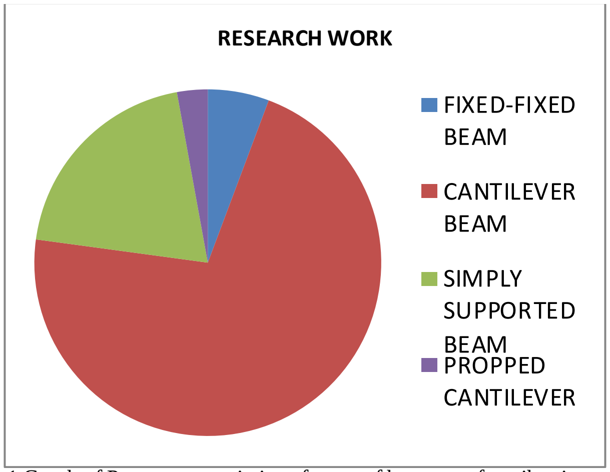


Fig.1 Graph of Percentage variation of types of beams use for vibration analysis of cracked beam

From above graph it is seen that most of the work is done using cantilever beam which is 69.44% according to literature. For simply supported beam only 19.44% work has been done which includes theoretical and FEM analysis only and little experimentation. In case of fixed-fixed beam 5.5% FEM work has been presented. Propped cantilever beam means a beam with one end fixed and the other is simply supported or roller supported is being a new topic for analysis. Only one analysis is done on this type of beam by FEM approach. Hence it is required to study the vibration analysis of cracked beam having one end fixed and the other is roller supported with nonlinear parameters for different crack locations, shapes and depth which are applicable for cases such as bridges, buildings.

III. THEORETICAL ANALYSIS OF TRANSVERSE VIBRATION OF FIXED-SIMPLY SUPPORTED BEAM [8]

Consider a beam with one end fixed and the other end is simply supported as shown in Fig.2

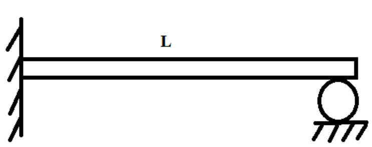


Fig.2 Beam with one end fixed and other is simply supported

From the Euler-Bernoulli's beam theory the relationship between bending moment and deflection is calculated as

$$M = EI\frac{\partial^2 y}{\partial x^2} \quad (1)$$

Where E is Young's Modulus in GPa and I is moment of inertia of the beam in m^4. The Equation (1) is based on the assumptions that the material is homogeneous, isotropic, obeys Hooke's law and the beam is straight and of uniform cross section. This Equation is valid only for small deflection and for beams that are long compared to cross sectional dimensions since the effects of shear deflection are neglected. The Equation of beam is

$$\frac{EI}{\rho A}\frac{\partial^4 y}{\partial x^4} + \frac{\partial^2 w}{\partial t^2} = 0 \quad (2)$$

Where, ρ is the mass density in kg/m^3 and A is cross sectional area of beam in m^2.

$$c^2\frac{\partial^4 y}{\partial x^4} + \frac{\partial^2 y}{\partial t^2} = 0 \quad (3)$$

Where,

$$C = \sqrt{\frac{EI}{\rho A}}$$

The solution of Equation (2) is to separate the variables one depends on position and another on time.

$$y = W(x)\,T(t) \quad (4)$$

By substituting Equation (4) in Equation (3), and simplifying, the Equation is

$$\frac{C^2}{w(x)}\frac{\partial^4 y}{\partial x^4} = -\frac{1}{T(t)}\frac{\partial^2 T(t)}{\partial t^2} \quad (5)$$

The Equation (5) can be written as two separate differential Equation

$$\frac{\partial^4 w}{\partial x^4} - \beta^2 W(x) = 0 \quad (5.a)$$

$$\frac{\partial^2 T}{\partial t^2} + \omega^2 T(t) = 0 \quad (5.b)$$

Where,

$$\beta^4 = \frac{\omega^2}{c^2} = \frac{\rho A\,\omega^2}{EI} \quad (6)$$

To find out the solution of Equation (5.a), consider the Equation

$$W(x) = C_1 \cos h\beta x + C_2 \sin h\beta x + C_3 \cos \beta x + C_4 \sin \beta x \quad (7)$$

In order to solve Equation (7) the following boundary conditions for Beam are needed:

The boundary condition we get,

For fixed end,

$$y(0,t) = 0 \quad , \frac{dy}{dx} = (0,t) \text{ and}$$

For simply supported end,

$$y(l,t) = 0 \quad , \frac{d^2y}{dx^2}(l,t) = 0 \quad (8)$$

Applying boundary conditions,

$$\tan \beta l = \tanh \beta l \quad (9)$$

$$\omega i = (\beta i L)^2 \sqrt{\frac{EI}{\rho A L^4}} \quad (10)$$

The first three roots of the Eq. (10) are shown in TABLE-I

TABLE-I
Value of Roots

Roots	β_i
1	3.9266
2	7.0686
3	10.2102

TABLE –II
Parameters for Beam

Parameters	Value	
Material	**SS**	**Al**
Total Effective Length	0.5 m	0.5 m
Width	0.025 m	0.025 m
Thickness	5 x 10^{-3}m	5 x 10-3m
Moment of Inertia	2.60x$10^{-10}$$m^4$	2.60x10-^{10}m^4
Young's Modulus	207 x10^9N/m^2	70 x 10^5N/m^2
Mass Density	7850 kg/m^3	2770 kg/m^3

Putting all required data in Equation (10) the three frequencies obtained are shown in TABLE-III.

TABLE-III
Theoretical Mode Shape Frequency (Hz)

Mode	Frequency in Hz for SS	Frequency in Hz for Al
1	29.27	28.220
2	117.09	114.487
3	263.45	257.596

IV.NUMERICAL ANALYSIS OF BEAM

Design of beam without crack is modeled in PRO-E software by using the properties given in TABLE II and import in ANSYS software.

A. Boundary conditions

The beam considered here has a fixed support at one end and simply supported at the other end of beam. For the fixed end all DOF are fixed as shown in fig 3.And at the simply supported end the displacement in Y direction is taken as free as shown in fig.4.

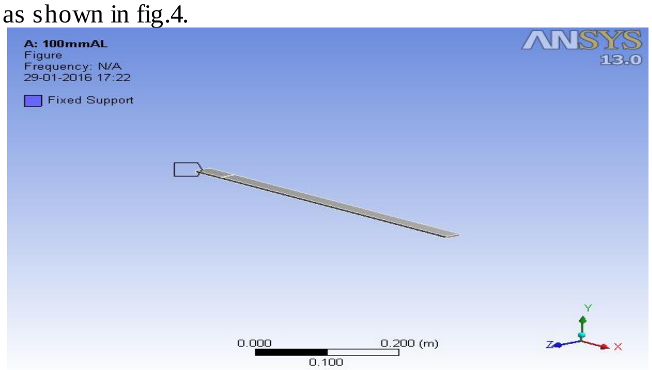


Fig.3 Fixed end beam

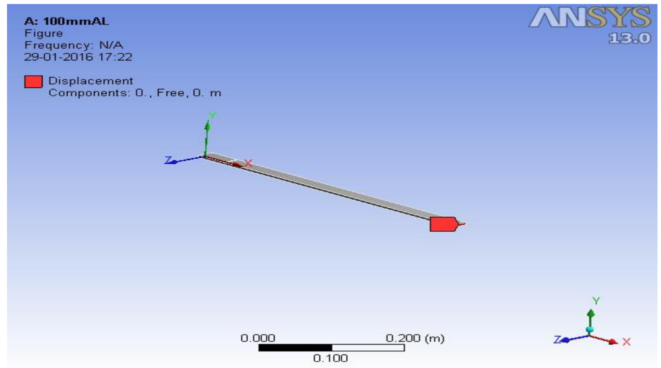


Fig.4 Simply supported end beam

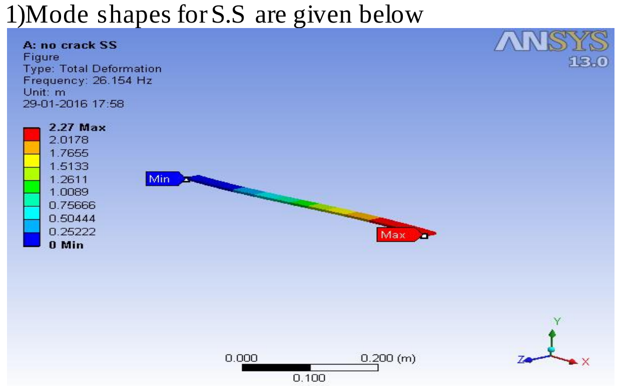
B. Vibration Analysis of Beam without Crack

The first step in the vibration analysis of the beam is to find its natural frequencies. In ANSYS, modal analysis used to find the Eigen natural frequencies. Initially the beam is taken without any crack. A minimum of first three mode shapes and natural frequencies are obtained and shown in Table IV. The lowest frequency of the beam is found to be 26.287 Hz for aluminum and 26.154 for structural steel .The mode shape and obtained lowest frequency for the beam without crack are shown in Fig.5 & Fig.6.

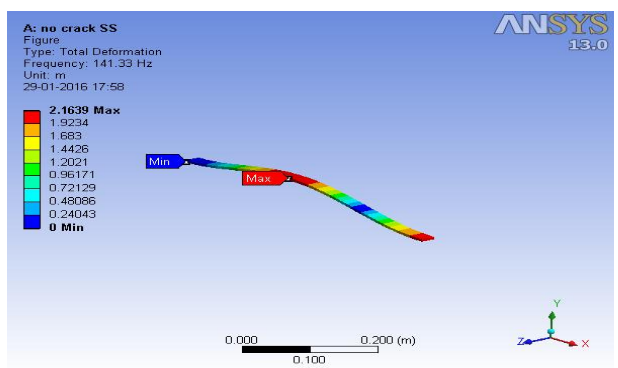
TABLE IV
Numerical Mode Shape Frequency (Hz) for without crack beam

Mode	Frequency [Hz]for SS	Frequency [Hz]for al
1.	26.154	26.287
2.	141.33	142.06
3.	348.85	350.67

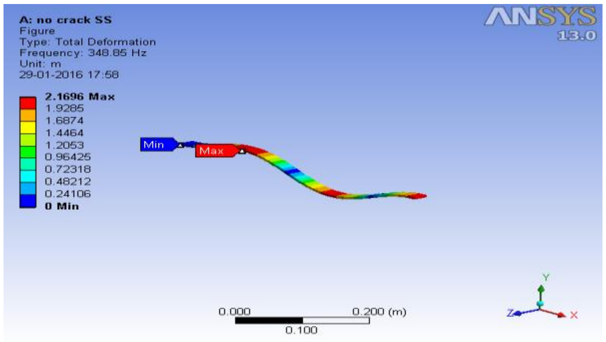
1)Mode shapes for S.S are given below



a)Mode Shape 1

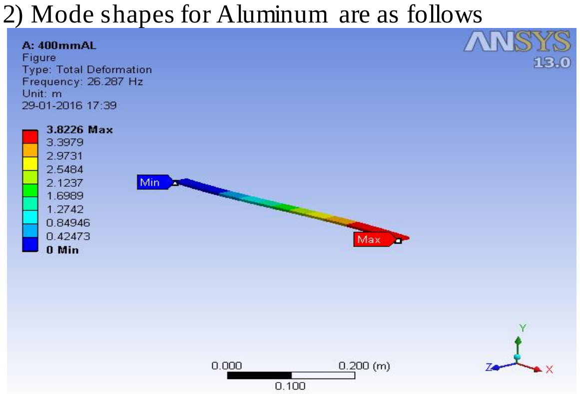


b) Mode Shape 2

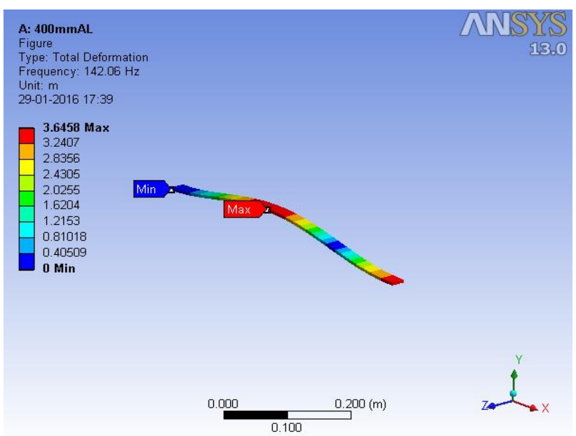


c) Mode Shape 3
Fig.5 Mode shape for Structural Steel Beam

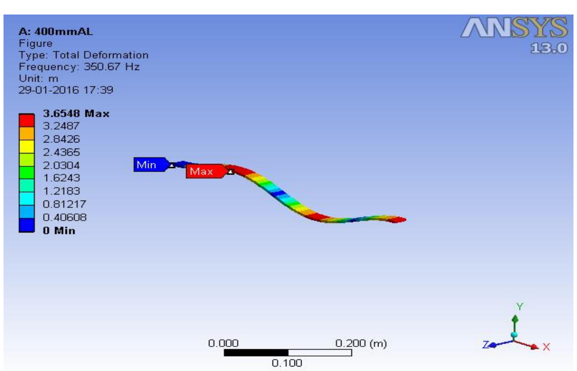
2) Mode shapes for Aluminum are as follows



a)Mode Shape 1



b) Mode Shape 2



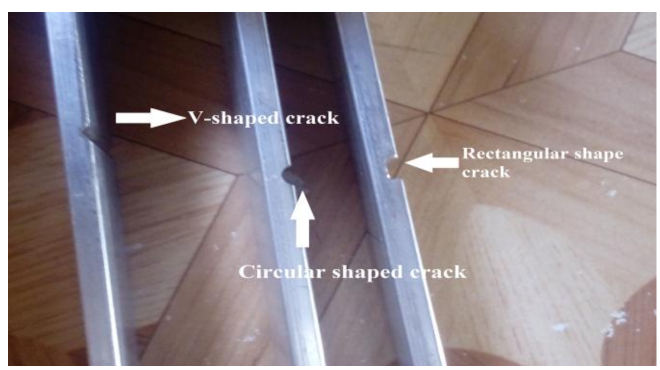
c) Mode Shape 3
Fig.6 Mode shape for Aluminum Beam

C. Design of Beam with Crack

For vibration analysis of a cracked beam, a triangular crack with a depth of 2 mm and width of 25 mm is considered. The initial position of the crack is taken at a location 100 mm from one end of the beam. Later, for comparative analysis the crack location is taken as 200mm, 250mm, 300m and 400mm. Then at particular location the crack depth is varied. Also at particular location the crack shape is varied and the results are compared.

a) Beam with different Crack (V-shaped) locations

b) Beam with different (V-shaped) Crack depth



c) Beam with different crack shape (i)Circular,(ii)V-shaped,(iii)Rectangular

Fig.7 Beam Specimen with Different crack locations, depth and shapes

V.EXPERIMENTAL STATIC ANALYSIS OF A BEAM WITH NONLINEAR PARAMETERS

Experimental Set Up For Load Deflection Curve:

Fig.8 shows a photograph of a system made up of a aluminum and structural steel beam of rectangular cross section fixed at one end and at the other end roller support is provided. The experimental measurements of the elastic curve of the beam as well as the vertical displacement at the middle of the beam are obtained on this set up. The large deflections of a simply supported beam are obtained by using ANSYS program; a comprehensive finite element package is used. Firstly Young's modulus of the material is obtained to do this experimental the values of the vertical displacements obtained at center, d_y, for different values of the concentrated load F applied at the center of the beam and obtained. The seven values for F: 1 to 7 kg to obtain the theoretical value of d_y for different values of E around the value of E=70 GPa (the typical value of Young's Modulus for Aluminum) and 200 GPa (the typical value of Young's Modulus for SS) using the ANSYS program. The Young's modulus E by comparing the experimentally measured displacements at the middle $d_{y.exp}$, (Fj), where j = 1, 2,…,J; J being the number of different external loads F considered (in our analysis J = 7), with the numerically calculated displacements d_y(E,Fj). We obtain the value of E for which the sum of the mean square root x^2 is minimum, where x^2 is given by the following equation [3]

$$x^2(E) = \sum_{j=1}^{J}\left[d_y\left(E,F_j\right) - \left(F_j\right)\right]^2 \quad (11)$$

Fig.8 Experimental setup to find out nonlinearities

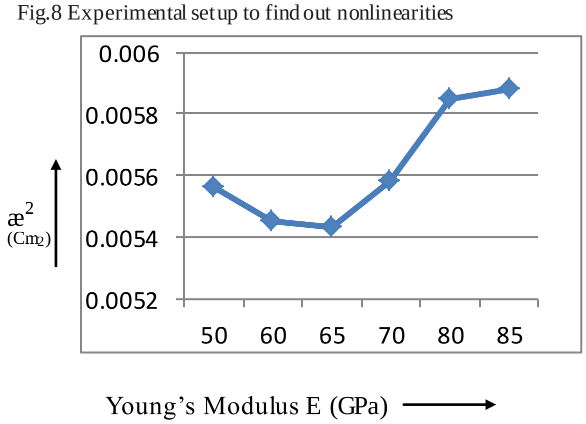


Fig.9 Calculated Values of $æ^2$ as a Function of E for Aluminum material

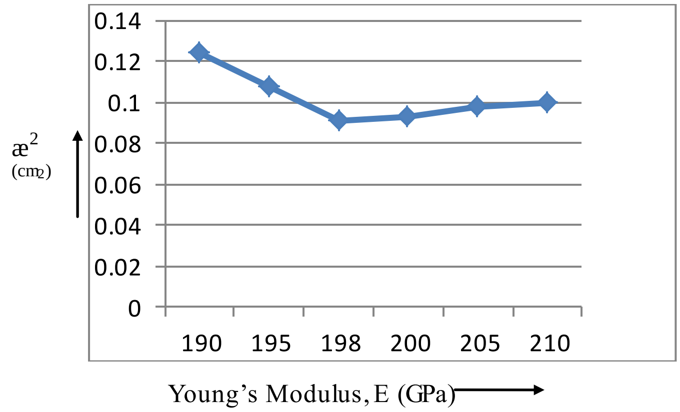


Fig.10 Calculated Values of x^2 as a Function of E for S.S. material

For aluminum we get E=65GPa and for structural steel it is E=198Gpa as shown in fig.9 and 10 respectively. By putting this new E we get the new natural frequency which is more accurate than previous as we are considering elasticity modulus as geometric nonlinearities.

VI. EXPERIENTAL VIBRATION ANALYSIS

Fig.12 & 13 shows a photograph of a system made up of a beam of rectangular cross section fixed at one end and at the other end roller support is provided. Fig.11 shows Experimental Set up for Static Analysis, in which proper connections of accelerometer, modal hammer, laptop and FFT Analyzer were made. Then by bump test various frequencies are obtained with the help of FFT Analyzer.

Fig.11 Experimental Setup for bump test

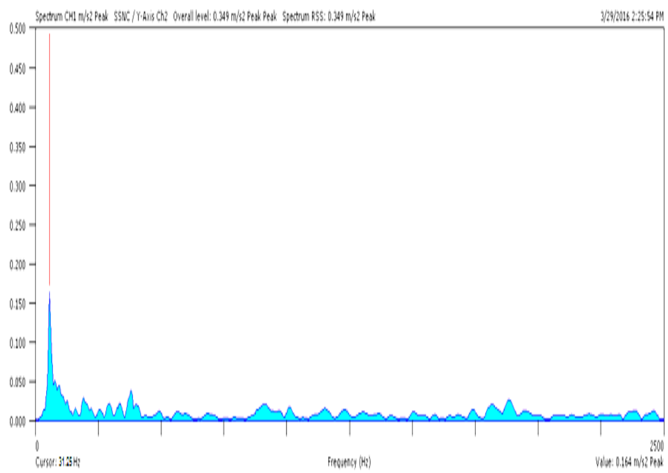
Fig.12Roller support


Fig.13 fixed support

Procedure for Free Vibrations

The connections i.e accelerometer, modal hammer, laptop and other power connections were made. The surface of the beam was cleaned for proper contact with the accelerometer. The accelerometer was then attached with the surface of the beam. We are using SKF FFT analyzer.

VII.RESULT & DISCUSSION


a) For without crack Structural Steel Beam

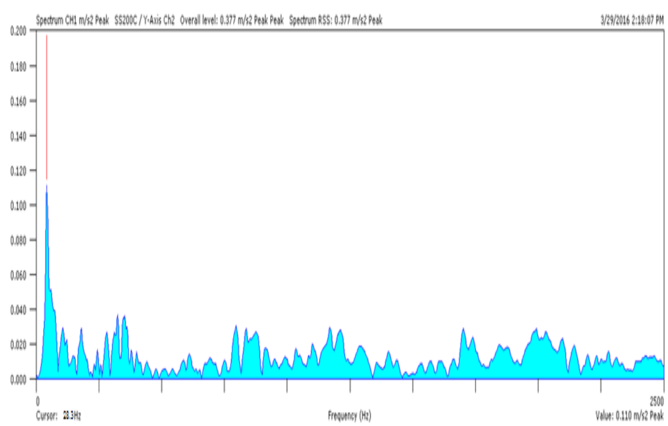

b) For Structural Steel cracked beam

Fig.14 Experimental frequency for SS beam

The natural frequency for structural steel beam without crack is 31.25Hz as shown in fig.14 a).For cracked beam the natural frequency decreases to 28.3Hz in presence of crack as shown in fig.14 b).

TABLE V

Frequency results using Numerical and Experimental modal analysis for different crack location

(a) For Structural Steel beam

Crack location from fixed end	Numerical Result (Hz)	FFT Result (Hz)	Numerical Result with nonlinearity (Hz)	% Error
No Crack	26.154	31.25	26.023	16.30
100	25.942	28.3	25.812	8.332
200	26.084	30.9	25.953	15.58
250	26.089	31.18	25.958	16.34
300	26.05	29.84	25.919	12.7
400	25.947	28.1	25.817	7.66

(b) For Aluminum beam

No Crack	26.287	32.37	25.151	18.8
100	25.895	28.125	24.77	7.92
200	26.132	28.56	25.003	8.5
250	26.203	31.25	25.072	16.17
300	26.165	28.79	25.035	9.11
400	26.064	28.24	24.938	7.70

TABLE VI

Frequency results using Numerical and Experimental modal analysis for different crack depth:

(a) For Structural Steel beam

Crack Depth in mm	Numerical Lowest frequency for SS(Hz)	Experimental Lowest frequency for SS in Hz	% Error
1	26.094	31.20	16.36
1.5	26.091	31.19	16.34
2	26.089	31.18	16.32

(b) For Aluminum beam

Crack Depth in mm	Numerical Lowest frequency for Aluminum(Hz)	Experimental Lowest frequency for Aluminum in Hz	% Error
1	26.207	31.32	16.32
1.5	26.205	31.28	16.22
2	26.203	31.25	16.15

TABLE VII

Frequency results using Numerical and Experimental modal analysis for different crack shapes:

(a) For Structural steel beam

Crack Shape	Numerical Lowest frequency for SS in Hz	Experimental Lowest frequency for SS in Hz	% Error
Circular	26.102	31.20	16.33
Rectangular	26.105	31.23	16.41
Triangular	26.089	31.18	16.32

(b) For Aluminum beam:

Crack Shape	Numerical Lowest frequency for Aluminum in Hz	Experimental Lowest frequency for AL in Hz	% Error
Circular	26.217	31.89	17.78
Rectangular	26.220	31.46	16.65
Triangular	26.203	31.25	16.15

From TABLE-V it is seen that as the crack location increase from fixed end the natural frequency increases from 28.3Hz to 31.18 Hz for location from 100mm to 250mm respectively and decreases to 28.1Hz at a location of 400mm for structural steel beam. For aluminum beam the natural frequency is 32.37Hz which is decreases to 28.125Hz in a presence of crack. In TABLE-VI shows the variation of crack depth. For 1mm crack depth the lowest natural frequency is 31.20Hz experimentally which is reduced to 31.18Hz at 2mm crack depth for S.S. In case of Aluminum the natural frequency decreases from 31.32 Hz to 31.25 Hz for 1mm crack depth to 2mm crack depth respectively. The percentage variation decreases by 2 percentages when the crack depth increases from 1mm to 2mm for both the materials. As the shape of crack changes the natural frequency changes. The triangular shape crack has lowest natural frequency, which is 26.089Hz for S.S and 26.203Hz for Aluminum beam than the circular and rectangular shape crack. The percentage error is largest for circular shape crack which is 17.78% for Aluminum and 16.33% for S.S.as shown in TABLE-VII.

VIII.CONCLUSIONS

Natural frequencies for beam without any crack are more than the cracked beam. Then the beam model with a triangular crack located initially at 100 mm, and then the crack locations are varied. The lowest frequencies found for each location of crack increases from 100 mm to 250 mm, which is the mid span of the beam, and decreases from there on. Further it can be found that at symmetric positions of the crack position of the beam the lowest fundamental frequencies have almost equal value. As the crack depth increases the natural frequency decreases. The natural frequency for triangular notch has smallest value for SS compared to rectangular and circular crack. The experimental analysis not only provides an understanding of geometric nonlinearities but also a better understanding of the basic concept of mechanics of materials.

ACKNOWLEDGMENT

Author would like to thank Dr.S.N.Shelke for guidance in this work. Also would like to thank mechanical department for providing facility to conduct experimentation in vibration laboratory.

REFERENCES

[1] Cook, R.D., and Young, W.C., Advanced Mechanics of Materials, Prentice Hall, New Jersey, 1999 .

[2] Nayfeh, A.H.andPai P.F., "Linear and Nonlinear Structure Mechanics", John Wiley & Sons, New York, 2003

[3] Tarsicio Belendez, "Numerical and Experimental Analysis Of A Cantilever Beam",Int.J.Engng Ed.Vol.19,No.6,pp.885-892,2003

[4] J. Fernad Ndez-Sad Ez, L. Rubio, C. Navarro," Approximate Calculation of The Fundamental Frequency For Bending Vibrations of Cracked Beams", Journal Of Sound And vibration (1999) 225(2), 345}352.

[5] D. Ravi Prasad and D.R. Seshu, "A study on Dynamic Characteristics of Structural Materials Using Modal Analysis", Asian Journal of Civil Engineering, Volume 9, Number 2, Pages 141-152, 2008

[6] Ranjan K. Behera, "Numerical and Experimental Verification of a Method for Prognosis of Inclined Edge Crack in Cantilever Beam Based on Synthesis of Mode Shapes", 2nd International Conference on Innovations in Automation and Mechatronics Engineering, ICIAME 2014

[7] P.Yamun1, K. Sambasivarao," Vibration Analysis Of Beam With Varying Crack Location", International Journal Of Engineering Research And General Science Volume 2, ISSN: 2091-2730,2014

[8] V.P.Singh, "Mechanical vibration",Dhanpat Rai & Co.

[9]Structural vibration frequency analysis of beams ©PDHengineer.com, a service mark of Decatur Professional Development LLC.S-3001 C1

Miss. Gangurde Priyanka P. born on 13 February 1993in India. She is a student of M.E. Mechanical (Design)of Gokhale Education Society's R.H. Sapat Collage of Engineering & Research Center,Nashik. She has completed her B.E. in Mechanical Engineering Pune University, K.K.W.COE, Nasik in the year 2014. She is currently working as an assistant professor in SCOE, Nasik. Her interesting subject is vibration.

Dr. Santosh Shelke born on 2 June 1977 in India. He has completed his Mechanical Engineering and post graduation in Design Engineering in Pune University, M.S. India in the years 1999 and 2005 respectively. Later he did his Ph.D at National Institute of technology, Warangal, A.P .on topic "Identification and Loss optimization in radial magnetic bearing using GA." He has published total 15 research papers on his research work including some reputed international journals such as Elsevier, Springer and IEEE etc. Author has total 16 years teaching experience and worked as HOD and Vice Principal at SVIT Nashik and presently working as Associate professor at R.H.sapat College of Engineering, Nasik. His research interest includes Design and optimization, advanced stress analysis and Advanced Mechanical Vibrations.