Experimental Study of Critical & Crossover Radius of Insulation

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Abstract: Critical radius is basically the thermo-physical parameter, which depends upon the conductivity of material and convective heat transfer coefficient. However, if it is desirable to decrease heat gain or heat loss, the critical radius only serves as a necessary condition, but it is not sufficient. To address design issues of thermal systems, the crossover radius is also one of the important parameter. At crossover radius the rate of heat transfer becomes equal to the uninsulated system. Hence it is not always customary to say that adding thickness beyond critical radius always decreases heat transfer, the farthest limit is defined by cross over radius.

Keywords: Critical radius; Crossover radius; Heat transfer rate; Thermal conductivity

I. INTRODUCTION

Use of thermal insulating materials is a very common practice for a wide range of applications. These are utilized for reducing heat gain such as in refrigeration piping, cryogenics, chilled water loops, electrical, transmission, refrigeration, industries and cryogenics etc. A thermal insulator is a poor conductor of heat and has a low thermal conductivity. Insulation is used in buildings and in manufacturing processes to prevent heat loss or heat gain. Although its primary purpose is an economic one, it also provides more accurate control of process temperatures and protection of personnel. (M. R. Kulkarni, 2004)

Critical radius of insulation is defined as it is the ratio of thermal conductivity of insulators to the heat transfer coefficient (k/h), or it is the radius at which the convection resistance almost negligible and the conduction resistance increases. The critical radius effect is an interesting phenomenon of heat transfer in insulated circular solids. Insulating a cylinder or sphere larger than the critical radius has expected effect of retarding heat loss. If a cylinder or sphere is a smaller than the critical radius, adding insulation will actually increase heat loss. Critical radius is independent of radius of circular solids. (Heat and mass transfer Book 4th Edition Y. A. Cengel, 2012)

When it is desirable to increase heat dissipation as in electrical transmission, critical radius can be used as radius of insulation for maximum heat dissipation. However, when it is desirable to decrease heat dissipation, critical radius is not always sufficient for such thermal systems the concept of crossover radius comes into play, and it is defined as the radius greater than the critical radius such that the heat transfer with the corresponding amount of insulating material is equal to that of the bare thermal system. (M. R. Kulkarni, 2003)

A thermal insulator is a poor conductor of heat and has a low thermal conductivity. Although its primary purpose is an economic one, it also provides more accurate control of process temperatures and protection of personnel. It prevents condensation on cold surfaces and the resulting corrosion. Thermal insulation delivers the following benefits: Reduces over-all energy consumption, offers better process control by maintaining process temperature, Prevents corrosion by keeping the exposed surface of a refrigerated system above dew point, Provides fire protection to equipment, Absorbs vibration. (Insulation materials and properties-section -2)

1.1- Critical radius of insulation for cylindrical & spherical system:
The critical radius of insulation dependent on thermal conductivity of the insulation (k) and the external convection heat transfer coefficient (h). The applications for a spherical surface which is coated with layers of different material or insulation, and in a radiation and convection environment, has wide application in industry. The critical thickness of insulation or coating is very important in many thermal applications. It is maximum thickness of insulator over the sphere at which we obtain maximum heat transfer rate. When radius of the insulator is increased beyond a certain limit the heat transfer rate instead of increasing start to decrease.
This is expected, since the heat transfer area $A$ is constant, and adding insulation always increases the thermal resistance of the wall without increasing the convection resistance. Adding insulation to a cylindrical pipe or a spherical shell, however, is a different matter. The additional insulation increases the conduction resistance of the insulation layer but decreases the convection resistance of the surface because of the increase in the outer surface area for convection. The heat transfer from the pipe may increase or decrease, depending on which effect dominates. (Heat and mass transfer Book 4th Edition Y. A. Cengel, 2012)

The rate of heat transfer from the insulated pipe to the surrounding air can be expressed as:

$$Q = \frac{T_1 - T_{\text{amb}}}{R_{\text{ins}} + R_{\text{conv}}} = \frac{T_1 - T_{\text{amb}}}{\frac{1}{h} + \frac{k}{2\pi r} + \frac{1}{2ml}} \quad \cdots (I)$$

The critical radius of insulation for a cylindrical body:

$$\left( r_c \right)_{\text{cylinder}} = \frac{k}{h} \quad \cdots (II)$$

The critical radius of insulation for a spherical shell:

$$\left( r_c \right)_{\text{sphere}} = \frac{2k}{h} \quad \cdots (III)$$

**II. EXPERIMENTAL SETUP**
The experimental setup consist of frame, thermocouples, temperature indicator, voltmeter, ammeter, copper cylinder, resistive heater, main switch, asbestos insulation. Frame length (Mild Steel) = 600mm, Frame height = 600mm, Frame width = 300mm. Supporting frame are placed on the main frame to support insulated cooper cylinder as well as resistive heater is placed inside of cooper cylinder as shown in fig3.

III. DESIGN PARAMETERS

3.1- Selection of material
Copper (Cylindrical pipe, k=385 Wm$^{-1}$K$^{-1}$)
Asbestos (Insulating material, k= 0.15 Wm$^{-1}$K$^{-1}$)
(Where, k thermal conductivity of materials is at room temperature)

3.2- Parameter used for design
Length, L= 0.6m
Radius of cylinder, $r_1 = 0.016$ m
Ambient temperature, $T_{amb} = 30^\circ$C
(Properties of air at 1 atm pressure, $T_{amb} = 30^\circ$C)
Density, $\rho = 1.164$ kgm$^{-3}$
Specific heat, $C_p=1007$ Jkg$^{-1}$K$^{-1}$
Mean film Temperature = $T_m$ $^\circ$C
Thermal conductivity, $k = 0.02588$ Wm$^{-1}$K$^{-1}$
Dynamic Viscosity, $\mu = 1.872X10^{-5}$ kgm$^{-1}$s$^{-1}$
Kinematic Viscosity, $\nu = 1.608X10^{-5}$ m$^2$s$^{-1}$
Prandtl Number, $P_r = 0.7282$
Outer radius, $r_2 = r_1 + t$ (t = thickness of insulation)

1. Mean film Temperature, $T_m$

\[ T_m = \frac{T_s + T_{amb}}{2} \]

2. Coefficient of Volumetric Expansion, $\beta$

\[ \beta = \frac{1}{T_m} \]

3. Raleigh number, $R_a D$

\[ R_a D = \frac{(g \beta (T_s - T_{amb}) \delta^3)}{\nu^2} \]

4. Empirical correction for Nusselt number of natural convection for horizontal cylinder $N_u$

\[ N_u = \frac{(0.6 + (0.388 \cdot \beta \cdot 6^{0.4}) \cdot \delta^{0.3})}{[1 + (0.555/P_r)^{0.24}]^{0.37}} \]

5. Convective heat transfer coefficient, $h$

\[ h = \frac{K \cdot N_u}{D} \]

6. Rate of heat transfer coefficient natural convection, $Q_{Conv}$

\[ Q_{Conv} = h \cdot A_d (T_s - T_{amb}) \]

7. Critical radius of insulation, $\tau_C$

\[ \tau_C = \frac{k}{h} \]
8. Rate of heat transfer for insulated cylinder, $Q$

\[
Q = \frac{T_1 - T_{\text{emb}}}{\ln(r_2/r_1)} + \frac{1}{h(2\pi r_2 L)} 
\]

### 3.3 Calculation for convective heat transfer coefficient

i. For $T_3 = 40^\circ C$

1. $T_m = \frac{T_2 + T_3}{2} = 35^\circ C = 30K$
2. $\beta = \frac{1}{300}$ K
3. $R_2 D = \frac{(981 \times 10^{-4} \times (40-35) \times 0.032^3 + 0.732^2)}{(1.60)^{3.5 - 0.5}} = 29393.196$
4. $N_u = \frac{(0.6 + (0.387 + 0.05939 \times 0.732^2) / 14)}{0.032 \times 5.702} = 5.703$
5. $h = \frac{0.032}{0.032} = 4.612 \text{ Wm}^{-2}\text{K}^{-1}$
6. $\tau_{cr} = \frac{\Delta T}{4.612}$

ii. For $T_3 = 50^\circ C$

$\tau_{cr} = 0.0274 \text{ m}$

iii. For $T_3 = 60^\circ C$

$\tau_{cr} = 0.0247 \text{ m}$

iv. For $T_3 = 70^\circ C$

$\tau_{cr} = 0.023 \text{ m}$

v. For $T_3 = 80^\circ C$

$\tau_{cr} = 0.023 \text{ m}$

Therefore Average Convective heat transfer coefficient

\[
\frac{4.612 + 5.474 + 6.184 + 6.501 + 6.887}{6} = 5.416
\]

### IV. EXPERIMENTAL INVESTIGATION

As per problem statement, setup is design to conduct an experiments on various insulating material and insulation thickness. Experiment have been conduct to analyze various thickness of insulations by varying voltage, current and temperature. Observations for different parameters are as follows:

<table>
<thead>
<tr>
<th>Radius $(\text{mm})$</th>
<th>$T_1$ $(^\circ \text{C})$</th>
<th>$T_2$ $(^\circ \text{C})$</th>
<th>$T_3$ $(^\circ \text{C})$</th>
<th>$T_4$ $(^\circ \text{C})$</th>
<th>$T_{\text{avg}}$ $(^\circ \text{C})$</th>
<th>$T_5$ $(^\circ \text{C})$</th>
<th>$T_6$ $(^\circ \text{C})$</th>
<th>$T_7$ $(^\circ \text{C})$</th>
<th>$T_8$ $(^\circ \text{C})$</th>
<th>$T_{\text{avg}}$ $(^\circ \text{C})$</th>
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</thead>
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<tr>
<td>19</td>
<td>35.6</td>
<td>36.2</td>
<td>36.2</td>
<td>36</td>
<td>36.62</td>
<td>32.6</td>
<td>33.8</td>
<td>35.4</td>
<td>35.4</td>
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<td>42.8</td>
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<td>43.4</td>
<td>42.7</td>
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<td>36</td>
<td>37.4</td>
<td>37.4</td>
<td>36.95</td>
</tr>
</tbody>
</table>

Table 1: Observation table

- $T_1, T_2, T_3, T_4$ = surface temperature of copper rod
- $T_5, T_6, T_7, T_8$ = outer temperature of insulation
- $R_1$ = radius of copper rod
- $R_2$ = radius of insulation
- $Q$ = heat transfer rate
- $T_{\text{avg}}$ = average outer temperature of insulation
- $T_{\text{amb}}$ = atmospheric temperature
- Insulation thickness: 3, 6, 9, 12, 15 mm
4.2 Calculations

\[ Q = \frac{T_s - T_{\text{amb}}}{\ln\left(\frac{r_2}{r_1}\right) + \frac{1}{h(2\pi r_2 L)}} \]

At \( r_2 = 19 \text{mm}, r_1=16\text{mm}, T_s = 34.25^\circ\text{C}, T_{\text{amb}} = 30^\circ\text{C}, L=0.6\text{m} \)

Heat transfer rate:

\[ Q = \frac{34.25 - 30}{\ln\left(\frac{19}{16}\right) + \frac{1}{2\pi \times 0.15 \times 0.6} + \frac{1}{2\pi \times 0.019 \times 0.6}} \]

\( Q_1 = 1.61 \text{ W} \)

Same procedure for other readings calculations and the heat transfer rate is:

\( Q_2 = 2.28 \text{ W} \)
\( Q_3 = 3.13 \text{ W} \)
\( Q_4 = 3.00 \text{ W} \)
\( Q_5 = 2.67 \text{ W} \)

V. RESULTS

Material with high thermal conductivity are good conductors of heat, whereas material with low thermal conductivity are good thermal insulators. When critical radius is less than the outer radius of insulation result in reduction of heat transfer in that segment of cylinder and when outer radius is less than critical radius of insulation result in increase of heat transfer. With surface radiation the critical radius is consistently lower.

![Fig4: Thickness of insulation Vs Heat transfer rate](image)

VI. CONCLUSION

By experimentation it is concluded that if the insulations thickness increases the heat transfer also increases up to a specific limit. After that at a one point of insulation thickness the heat transfer rate is at its maximum limit and then decreases. This point of insulation thickness is called critical point of insulation. After critical point heat transfer rate reduces this goes to crossover point of insulation or heat transfer rate is negligible. In other words the cross over radius gives an idea up to where we add the insulation thickness to reduce the heat loss. Critical thickness are dependent on shape and size of object.

REFERENCES