Estimation of Aerodynamic Derivatives of A Wedge in Hypersonic Flow

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ABSTRACT

In the present paper a theory for 2-D slender bodies at high angles of attack in hypersonic flow has been developed. This theory has been applied to a sharp thick wedge with attached shock case. Using the theory a relation for a piston moving in a cylinder at any velocity and relations for stiffness and damping derivatives are obtained for zero incidence of the wedge and it is found to be dependent on flight Mach number and wedge semi vertex angle. Results show that the stiffness and damping derivatives increase with the increase in the semi vertex angle. The present method includes the thin wedge case as well, which was covered by Lighthill piston theory, and applies only for small amplitude and low reduced frequency case. Effect of viscosity and secondary wave reflections have been neglected. The results are obtained for wedges of different semi-vertex angles and Mach numbers for a perfect gas.

Keywords—Mach Number, Angle of attack, hypersonic Flow, stability derivative.

INTRODUCTION

The idea of hypersonic similitude is due to Tsien [1], who investigated the two-dimensional and axi-symmetric irrotational equations of motion. Equivalence of a steady hypersonic flow on a slender body with an unsteady flow in one fewer space dimensions was pointed out by Hayes [2]. Unsteady supersonic flow has been well studied by using the potential theory. Unsteady hypersonic flow has been studied by many researchers using unsteady analogues of either the shock expansion theory or the tangent wedge approximation [3,4].

For an oscillating wedge, more theoretical studies have been made, such a theory of oscillating aero foils at high Mach number is developed by Lighthill [6]. Here the pitching oscillation is taken into consideration. Lighthill uses Hayes [3,4] results that to a good approximation, any plane slab of fluid, initially perpendicular to the undisturbed flow, remains so as it is swept downstream and moves in its own plane under the laws of ‘one dimensional unsteady motion’. A parameter τ is introduced, whose purpose will be to serve as a measure of the maximum inclination angle of Mach waves in the flow field. Here it is assumed that M₁ ≤ τ and τ is of the order of maximum deflection of a stream line. So in the flow past an aerofoil at high Mach number, the perturbations and gradients are much larger in the lateral direction than those in the axial direction.

Sychev’s law of plane sections for 3-D slender bodies at high angles of attack in hypersonic flow [5] has been discussed. Based on this a new piston analogy has been given for a 2-D slender body at high angle of attack in hypersonic flow. The new piston analogy has been first applied to get the stiffness and damping coefficients of an oscillating flat plate at high angle of incidence for which the bow shock is attached. Next assuming that two flat plates have formed a sharp wedge, closed form formulae for stiffness and damping coefficients have been obtained for it. Viscous effect and wave reflections are not taken into account in this development. Results are obtained for
hypersonic flow of a perfect gas over the wedges of different semi-vertex angles.

II. ANALYSIS

Let the flat plate aerofoil be of length $L$ at mean angle of incidence $\theta$ and oscillating in pitch with small amplitude about a pivot point $O_1$ at a distance $x_0$ from its apex. The angle of attack at any instant is $\alpha$. So the velocity of the infinitesimal piston (an infinitesimal part of the aerofoil) at point $x$, can be written as

$$U_p = U_x \sin \alpha + \dot{a}(x - x_0)$$  \hspace{1cm} (1)

To relate the piston velocity with pressure on the face of piston, Lighthill\cite{4} suggested the use of 3 terms in the isentropic expression for the pressure on a piston as a power series in its velocity. So a condition that the piston velocity should be less than or equal to free stream sound velocity is imposed to satisfy isentropic assumption. This is consistent with hypersonic small disturbance theory on which Lighthill’s piston analogy is based. Then the Pressure ratio can be written as

$$\frac{P_x}{\rho_0} = 1 + A \left( \frac{U_x}{a_s} \right)^{2} \chi 2 + B \left( \frac{U_x}{a_s} \right)^{2} \chi 4$$  \hspace{1cm} (2)

Where, $A = \frac{y(y+1)}{4}$ \hspace{1cm} $B = \left( \frac{4}{y+1} \right)^{2}$

The nose down moment,

$$-m = \int_{0}^{L}(x - x_0)P_2 \, dx$$ \hspace{1cm} (3)

Of the load distribution about the axis $x = x_0$. $P_2$ is the pressure on the windward surface. The pressure on the leeward surface is taken to be zero.

Thus the aerodynamic stiffness derivative is,

$$-c_{m\alpha} = (y + 1) \tan \theta \left[ 2 + \frac{M_s^2 \sin^2 \theta + \left( \frac{4}{y+1} \right)^{2}}{M_s \sin \theta} \right] \left( \frac{1}{2} - h_0 \cos^2 \theta \right)$$  \hspace{1cm} (4)

And the aerodynamic damping derivative is,

$$-c_{m\theta} = (y + 1) \frac{\tan \theta}{\cos \theta} \left[ 2 + \frac{M_s^2 \sin^2 \theta + \left( \frac{4}{y+1} \right)^{2}}{M_s \sin \theta} \right] \left( \frac{1}{3} - \frac{h_0 \cos^2 \theta + h_0^2 \cos^4 \theta}{h} \right)$$  \hspace{1cm} (5)

III. RESULTS AND DISCUSSIONS

![Graph for stiffness and damping derivative M=5, $\theta=5$ with pivot position](image1)

Fig 1: Variation of Stiffness and Damping for $M=5$, $\theta=5$ with pivot position

![Graph for stiffness and damping derivative M=5, $\theta=10$ with pivot position](image2)

Fig 2: Variation of Stiffness and Damping for $M=5$, $\theta=10$ with pivot position

![Graph for stiffness and damping derivative M=5, $\theta=15$ with pivot position](image3)

Fig 3: Variation of Stiffness and Damping for $M=5$, $\theta=15$ with pivot position
Figs. 1, 2, 3, 4, 5, and 6 show the variation of Stiffness and damping derivatives for Mach number 5 and different angle of attack from 5 degrees to 30 degrees. From the figure 1 and 2 it is seen that as the semi vertex angle increases the stiffness derivative values increases in magnitude. It is also seen that due to the increase in the semi vertex angle of the wedge there is substantial increase in the plan form area of the wedge which results in 54% increase in the stiffness derivative and 44% increase in the damping derivative. From figure 3 and 4 it is seen that when semi vertex angle is increased to 15 and 20 degrees respectively results in 29% and 40% increment in stiffness and damping derivatives. Similarly when the semi vertex angle is further increased to 25 and 30 degrees results in 22% and 39% increase in the stability derivatives in pitch and shown in figures 5 and 6. The stiffness derivative is the maximum at the $h = 0$, that is at the nose of the wedge, since at this point the moment arm will be the maximum and then it decreases linearly with the pivot position. Even in case of damping derivative we can see that magnitude increases with increase in semi vertex angle and the minima shifts towards the trailing edge of the wedge. From the figures it is also observed that with the increase in the semi vertex angle of the wedge the center of pressure also shifts towards the trailing edge, the reason for this shift is due to the increase in the plan form area of the wedge and more so the major portion of plan form area is transferred towards the trailing edge leading to maximum plan form area available for the case at $h = 1$.

IV. CONCLUSION

All estimations are based on in viscous flow past sharp wedges. It is seen that the Stiffness and Damping derivative decreases with the increase in the Mach number. Further it increases with the increase in semi vertex angle. The Stiffness derivative decreases with pivot position whereas, in case of damping derivative first it decreases then attains a minima which is position of center of pressure then increases. Secondary wave reflections generated from the body and reflected from the bow shock are neglected. Hui’s has mentioned another set of waves generated due to the unsteady motion of the bow shock. These waves are also not considered in the present case. Finally, perturbation, tangential to the wedge surface is also neglected as per Sychev law of plane sections.

REFERENCES


